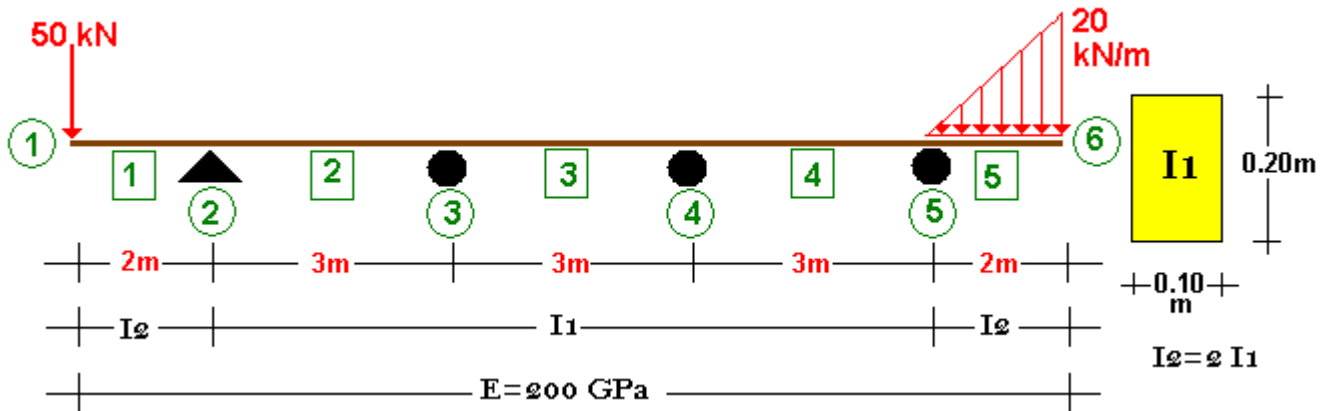
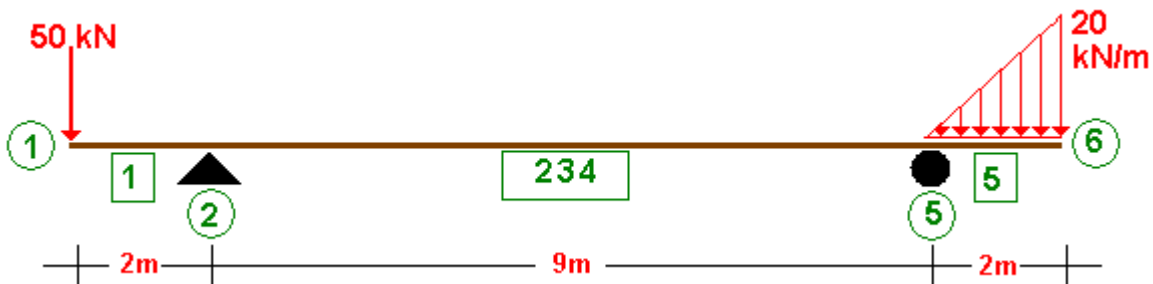


Encuentre las reacciones por el método de las Flexibilidades, diagrama de fuerza cortante y momento flector de la siguiente viga hiperestática.



$$\#1: \left[I_1 := \frac{1}{12} \cdot 0.1 \cdot 0.2^3, I_2 := 2 \cdot I_1, E := 200 \cdot 10^6 \right]$$

Flexibilización de la estructura



Cálculo de reacciones de la viga flexibilizada

$$\#2: \left[\begin{array}{l} R_{f2} + R_{f5} = 50 + \frac{20 \cdot 2}{2} \\ R_{f5} \cdot 9 + 50 \cdot 2 - \frac{20 \cdot 2}{2} \cdot \left(9 + \frac{2}{3} \cdot 2 \right) \end{array} \right]$$

$$\#3: \left[R_{f2} := \frac{1570}{27}, R_{f5} := \frac{320}{27} \right]$$

$$\#4: [R_{f2} := 58.14814814, R_{f5} := 11.85185185]$$

Momentos flectores de la viga flexibilizada $MF(x)$

$$\#5: MF_{f1}(x) := -50 \cdot x$$

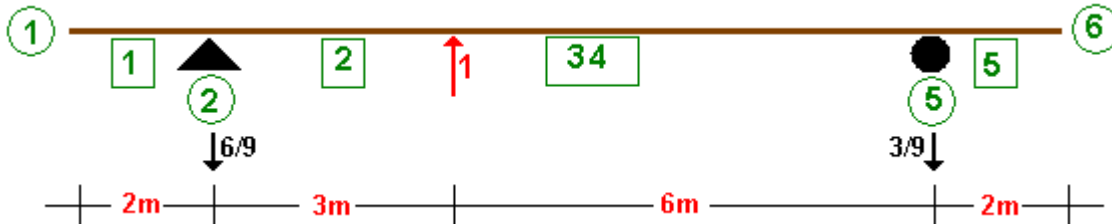
$$\#6: MF_{f234}(x) := -50 \cdot x + R_{f2} \cdot (x - 2)$$

#7:
$$\text{MFf234}(x) := 8.148148148 \cdot x - 116.2962962$$

#8:
$$\text{MFf5}(x) := -50 \cdot x + \text{Rf2} \cdot (x - 2) + \text{Rf5} \cdot (x - 11) - \frac{1}{2} \cdot 10 \cdot (x - 11)^2 \cdot \frac{1}{3} \cdot (x - 11)$$

#9:
$$\text{MFf5}(x) := -1.666666666 \cdot x^3 + 55 \cdot x^2 - 585 \cdot x + 1971.666666$$

Carga unitaria ficticia en el nudo 3 con las reacciones en el nudo 2 y 5



Momentos flectores ficticios del estado (I) $\text{mfI}(x)$

#10:
$$\text{mfI1}(x) := 0$$

#11:
$$\text{mfI2}(x) := -\frac{6}{9} \cdot (x - 2)$$

#12:
$$\text{mfI2}(x) := 1.333333333 - 0.666666666 \cdot x$$

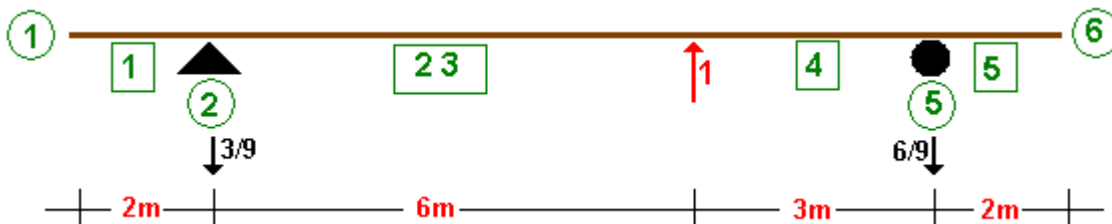
#13:
$$\text{mfI34}(x) := -\frac{6}{9} \cdot (x - 2) + 1 \cdot (x - 5)$$

#14:
$$\text{mfI34}(x) := 0.333333333 \cdot x - 3.666666666$$

#15:
$$\text{mfI5}(x) := -\frac{6}{9} \cdot (x - 2) + 1 \cdot (x - 5) - \frac{3}{9} \cdot (x - 11)$$

#16:
$$\text{mfI5}(x) := 0$$

Carga unitaria ficticia en el nudo 4 con las reacciones en el nudo 2 y 5



Momentos flectores ficticios del estado (II) $\text{mfII}(x)$

#17:
$$\text{mfII1}(x) := 0$$

$$\#18: \text{mfII23}(x) := -\frac{3}{9} \cdot (x - 2)$$

$$\#19: \text{mfII23}(x) := 0.6666666666 - 0.3333333333 \cdot x$$

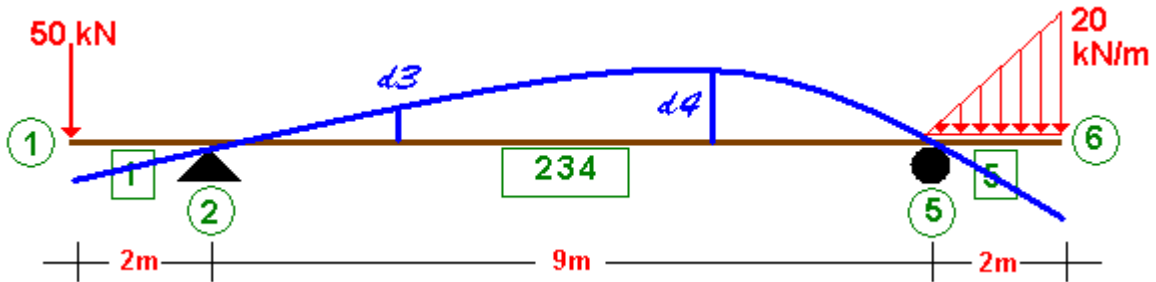
$$\#20: \text{mfII4}(x) := -\frac{3}{9} \cdot (x - 2) + 1 \cdot (x - 8)$$

$$\#21: \text{mfII4}(x) := 0.6666666666 \cdot x - 7.3333333333$$

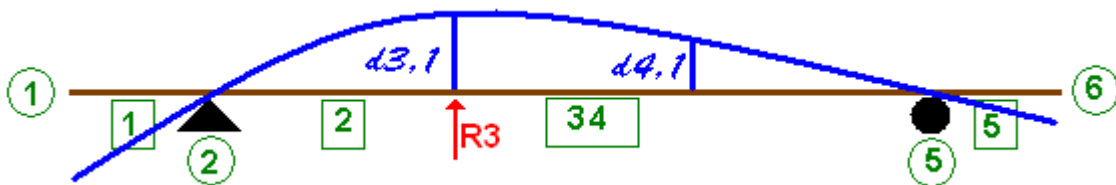
$$\#22: \text{mfII5}(x) := -\frac{3}{9} \cdot (x - 2) + 1 \cdot (x - 8) - \frac{6}{9} \cdot (x - 11)$$

$$\#23: \text{mfII5}(x) := 0$$

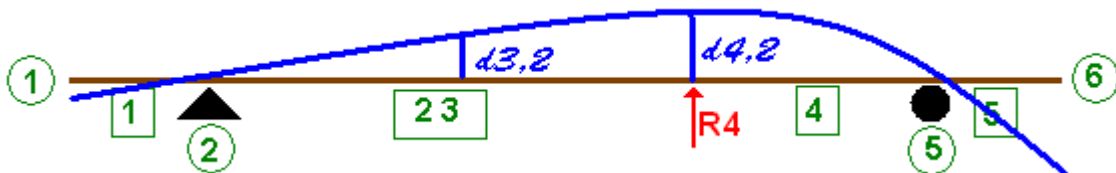
Deflexiones en la viga flexibilizada df



Deflexiones si se carga con R3



Deflexiones si se carga con R4



Cálculo de las deflexiones en la viga flexibilizada y con cargas originales:

$$\#24: \quad df3 := \frac{1}{E \cdot I2} \cdot \int_0^2 MFf1(x) \cdot mfI1(x) \, dx + \frac{1}{E \cdot I1} \cdot \int_2^5 MFf234(x) \cdot mfI2(x) \, dx + \frac{1}{E \cdot I1} \cdot \int_5^{11} MFf234(x) \cdot mfI34(x) \, dx + \frac{1}{E \cdot I2} \cdot \int_{11}^{13} MFf5(x) \cdot mfI5(x) \, dx$$

$$\#25: \quad df3 := \frac{1}{E \cdot I2} \cdot \int_0^2 (-50 \cdot x) \cdot 0 \, dx + \frac{1}{E \cdot I1} \cdot \int_2^5 (8.148148148 \cdot x - 116.2962962) \cdot (1.333333333 - 0.6666666666 \cdot x) \, dx + \frac{1}{E \cdot I1} \cdot \int_5^{11} (8.148148148 \cdot x - 116.2962962) \cdot (0.3333333333 \cdot x - 3.666666666) \, dx + \frac{1}{E \cdot I2} \cdot \int_{11}^{13} (-1.666666666 \cdot x^3 + 55 \cdot x^2 - 585 \cdot x + 1971.666666) \cdot 0 \, dx$$

$$\#26: \quad df3 := 0 + \frac{1}{E \cdot I1} \cdot \int_2^5 (8.148148148 \cdot x - 116.2962962) \cdot (1.333333333 - 0.6666666666 \cdot x) \, dx + \frac{1}{E \cdot I1} \cdot \int_5^{11} (8.148148148 \cdot x - 116.2962962) \cdot (0.3333333333 \cdot x - 3.666666666) \, dx + 0$$

$$\#27: \quad df3 := 0 + \frac{1}{E \cdot I1} \cdot 251.1111111 + \frac{1}{E \cdot I1} \cdot 355.5555555 + 0$$

$$\#28: \quad df3 := 0 + (7.5 \cdot 10^{-5}) \cdot 251.1111111 + (7.5 \cdot 10^{-5}) \cdot 355.5555555 + 0$$

$$\#29: \quad df3 := 0 + 0.01883333333 + 0.02666666666 + 0$$

$$\#30: \quad df3 := 0.0455$$

$$\#31: \quad df4 := \frac{1}{E \cdot I2} \cdot \int_0^2 MFf1(x) \cdot mfIII1(x) \, dx + \frac{1}{E \cdot I1} \cdot \int_2^8 MFf234(x) \cdot mfIII23(x) \, dx +$$

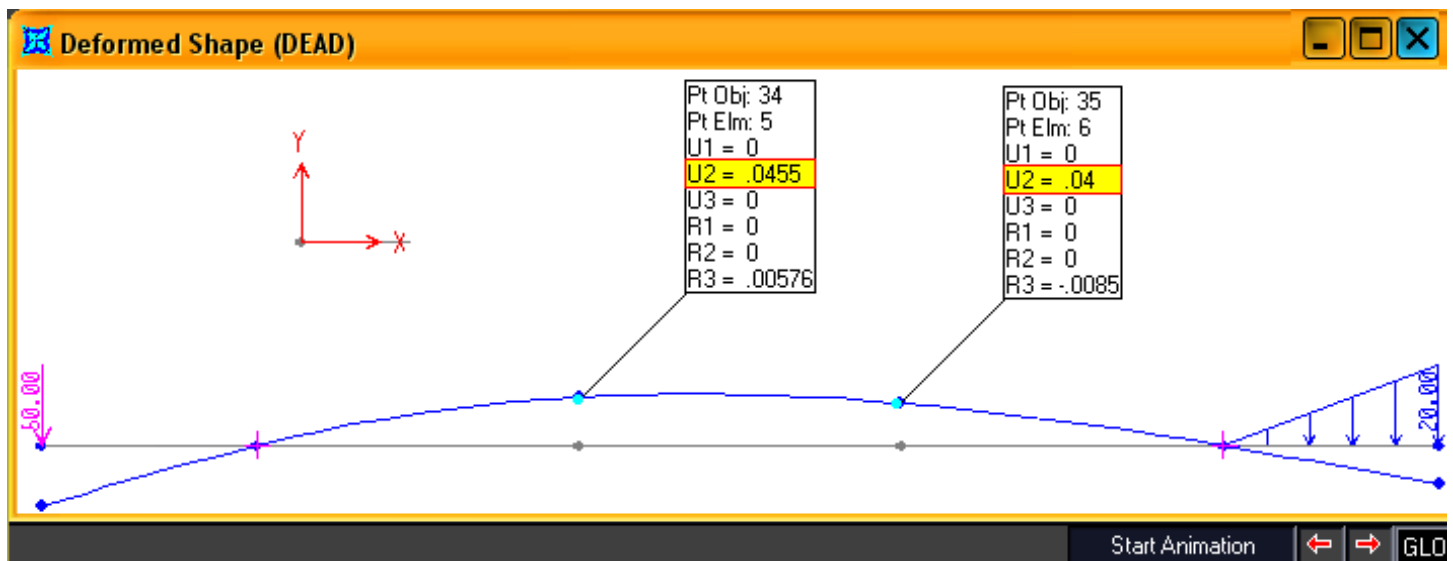
$$\frac{1}{E \cdot I1} \cdot \int_8^{11} MFf234(x) \cdot mfIII4(x) \, dx + \frac{1}{E \cdot I2} \cdot \int_{11}^{13} MFf5(x) \cdot mfIII5(x) \, dx$$

$$\#32: \quad df4 := 0 + \frac{1}{E \cdot I1} \cdot \int_2^8 MFf234(x) \cdot mfIII23(x) \, dx + \frac{1}{E \cdot I1} \cdot \int_8^{11} MFf234(x) \cdot mfIII4(x) \, dx + 0$$

$$\#33: \quad df4 := 0 + 0.0303333333333 + 0.0096666666666 + 0$$

$$\#34: \quad df4 := 0.04$$

Comparación de estos resultados con resultados de programas de computador comerciales (Sap2000 en este caso):



Cálculo de las deflexiones con la viga cargada con 1 kN en R3:

$$\#35: \quad d3I := \frac{1}{E \cdot I2} \cdot \int_0^2 mfI1(x) \cdot mfI1(x) \, dx + \frac{1}{E \cdot I1} \cdot \int_2^5 mfI2(x) \cdot mfI2(x) \, dx + \frac{1}{E \cdot I1} \cdot \int_5^{11}$$

$$mfI34(x) \cdot mfI34(x) \, dx + \frac{1}{E \cdot I2} \cdot \int_{11}^{13} mfI5(x) \cdot mfI5(x) \, dx$$

$$\#36: \quad d3I := 0 + 0.0003 + 0.0006 + 0$$

$$\#37: \quad d3I := 0.0009$$

$$\begin{aligned} \#38: \quad d4I &:= \frac{1}{E \cdot I2} \cdot \int_0^2 m_{fII1}(x) \cdot m_{fIII1}(x) \, dx + \frac{1}{E \cdot I1} \cdot \int_2^5 m_{fII2}(x) \cdot m_{fIII23}(x) \, dx + \frac{1}{E \cdot I1} \cdot \int_5^8 \\ & m_{fII34}(x) \cdot m_{fIII23}(x) \, dx + \frac{1}{E \cdot I1} \cdot \int_8^{11} m_{fII34}(x) \cdot m_{fIII4}(x) \, dx + \frac{1}{E \cdot I2} \cdot \int_{11}^{13} \\ & m_{fII5}(x) \cdot m_{fIII5}(x) \, dx \end{aligned}$$

$$\#39: \quad d4I := 0 + 0.00015 + 0.0004875 + 0.00015 + 0$$

$$\#40: \quad d4I := 0.0007875$$

$$\begin{aligned} \#41: \quad d3II &:= \frac{1}{E \cdot I2} \cdot \int_0^2 m_{fIII1}(x) \cdot m_{fII1}(x) \, dx + \frac{1}{E \cdot I1} \cdot \int_2^5 m_{fIII23}(x) \cdot m_{fII2}(x) \, dx + \frac{1}{E \cdot I1} \cdot \int_5^8 \\ & m_{fIII23}(x) \cdot m_{fII34}(x) \, dx + \frac{1}{E \cdot I1} \cdot \int_8^{11} m_{fIII4}(x) \cdot m_{fII34}(x) \, dx + \frac{1}{E \cdot I2} \cdot \int_{11}^{13} \\ & m_{fIII5}(x) \cdot m_{fII5}(x) \, dx \end{aligned}$$

Cálculo de las deflexiones con la viga cargada con 1 kN en R4:

$$\#42: \quad d3II := 0 + 0.00015 + 0.0004875 + 0.00015 + 0$$

$$\#43: \quad d3II := 0.0007875$$

$$\begin{aligned} \#44: \quad d4II &:= \frac{1}{E \cdot I2} \cdot \int_0^2 m_{fIII1}(x) \cdot m_{fIII1}(x) \, dx + \frac{1}{E \cdot I1} \cdot \int_2^8 m_{fIII23}(x) \cdot m_{fIII23}(x) \, dx + \\ & \frac{1}{E \cdot I1} \cdot \int_8^{11} m_{fIII4}(x) \cdot m_{fIII4}(x) \, dx + \frac{1}{E \cdot I2} \cdot \int_{11}^{13} m_{fIII5}(x) \cdot m_{fIII5}(x) \, dx \end{aligned}$$

$$\#45: \quad d4II := 0 + 0.0006 + 0.0003 + 0$$

$$\#46: \quad d4II := 0.0009$$

Plantemiento de las ecuaciones del método de las flexibilidades:

$$\#47: \begin{bmatrix} df3 + d3I \cdot R3 + d3II \cdot R4 = 0 \\ df4 + d4I \cdot R3 + d4II \cdot R4 = 0 \end{bmatrix}$$

$$\#48: \begin{bmatrix} 0.0455 + 0.0009 \cdot R3 + 0.0007875 \cdot R4 = 0 \\ 0.04 + 0.0007875 \cdot R3 + 0.0009 \cdot R4 = 0 \end{bmatrix}$$

$$\#49: [R3 := -49.77777777, R4 := -0.8888888888]$$

Cálculo de las reacciones R2 y R5 por las ecuaciones de la estática

$$\#50: \begin{bmatrix} R2 + R3 + R4 + R5 = 50 + \frac{20 \cdot 2}{2} \\ R3 \cdot 3 + R4 \cdot 6 + R5 \cdot 9 + 50 \cdot 2 - \frac{20 \cdot 2}{2} \cdot \left(9 + \frac{2}{3} \cdot 2 \right) = 0 \end{bmatrix}$$

$$\#51: [R2 := 91.62962962, R5 := 29.03703703]$$

Ecuaciones del diagrama de momento flector de la estructura original con las incógnitas resueltas:

$$\#52: MF1(x) := -50 \cdot x$$

$$\#53: MF2(x) := -50 \cdot x + R2 \cdot (x - 2)$$

$$\#54: MF2(x) := 41.62962962 \cdot x - 183.2592592$$

$$\#55: MF3(x) := -50 \cdot x + R2 \cdot (x - 2) + R3 \cdot (x - 5)$$

$$\#56: MF3(x) := 65.62962962 - 8.148148148 \cdot x$$

$$\#57: MF4(x) := -50 \cdot x + R2 \cdot (x - 2) + R3 \cdot (x - 5) + R4 \cdot (x - 8)$$

$$\#58: MF4(x) := 72.74074074 - 9.037037037 \cdot x$$

$$\#59: MF5(x) := -50 \cdot x + R2 \cdot (x - 2) + R3 \cdot (x - 5) + R4 \cdot (x - 8) + R5 \cdot (x - 11) - \frac{1}{2} \cdot 10 \cdot (x - 11)$$

$$11) \cdot \frac{2}{3} \cdot (x - 11)$$

$$\#60: MF5(x) := -1.666666666 \cdot x^3 + 55 \cdot x^2 - 585 \cdot x + 1971.666666$$

Puntos importantes para dibujar el diagrama de momento flector:

$$\#61: \left[\begin{array}{ccc} \text{MF1}(0) = 0 & \text{MF1}(1) = -50 & \text{MF1}(2) = -100 \\ \text{MF2}(2) = -100 & \text{MF2}(3.5) = -37.55555555 & \text{MF2}(5) = 24.88888888 \\ \text{MF3}(5) = 24.88888888 & \text{MF3}(6.5) = 12.66666666 & \text{MF3}(8) = 0.4444444444 \\ \text{MF4}(8) = 0.4444444444 & \text{MF4}(9.5) = -13.11111111 & \text{MF4}(11) = -26.66666666 \\ \text{MF5}(11) = -26.66666666 & \text{MF5}(12) = -8.33333333 & \text{MF5}(13) = 0 \end{array} \right]$$

Diagrama de momento flector:

