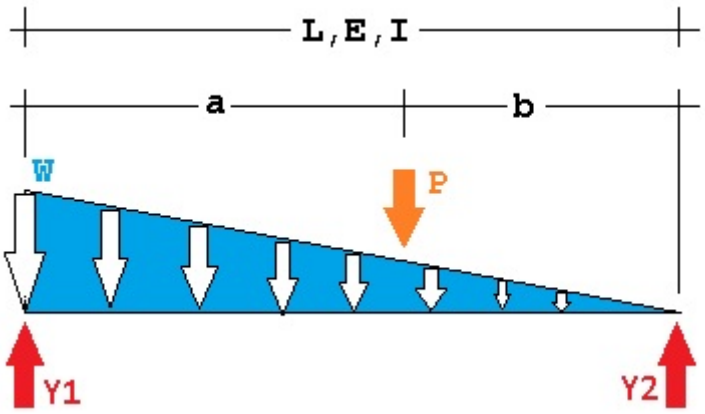


Utilizando el Método del Área del Diagrama de Momento flector, se calculará la curva elástica de la viga indicada arriba

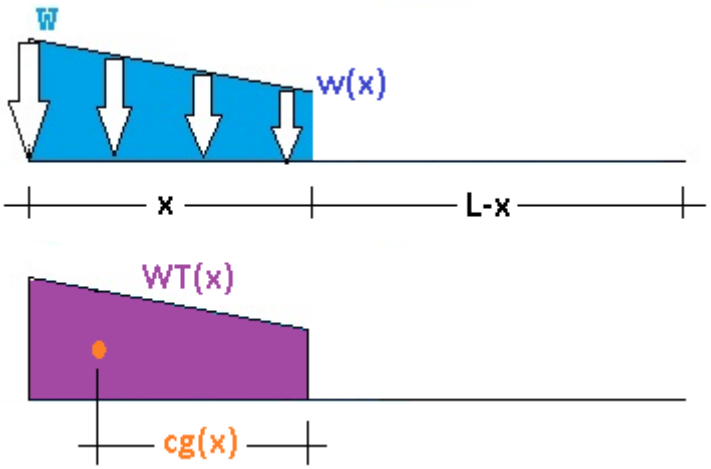
#1: [CaseMode := Sensitive, InputMode := Word, w(x) :=]

Cálculo de la reacciones:

#2: $$\left[\begin{array}{c} Y1 + Y2 = \frac{W \cdot L}{2} + P \\ Y2 \cdot L - P \cdot a - \frac{W \cdot L}{2} \cdot \frac{L}{3} = 0 \end{array}\right]$$

#3: $$\left[Y1 = \frac{P \cdot (L - a)}{L} + \frac{L \cdot W}{3} \wedge Y2 = \frac{P \cdot a}{L} + \frac{L \cdot W}{6}\right]$$

Valores de la carga en cualquier punto x, w(x), de la carga total hasta cualquier punto x, WT(x), y del centroide medido desde el punto x, cg(x):



#4: $$\frac{W}{L} = \frac{w(x)}{L - x}$$

#5: $$w(x) := \frac{W\cdot(L - x)}{L}$$

#6: $$WT(x) := \frac{W + w(x)}{2}\cdot x$$

#7: $$WT(x) := \frac{W + \frac{W\cdot(L - x)}{L}}{2}\cdot x$$

#8: $$WT(x) := \frac{W\cdot x\cdot(2\cdot L - x)}{2\cdot L}$$

#9: $$cg(WT(x)) = w(x)\cdot x\cdot\frac{x}{2} + \frac{(W - w(x))\cdot x}{2}\cdot\frac{2}{3}\cdot x$$

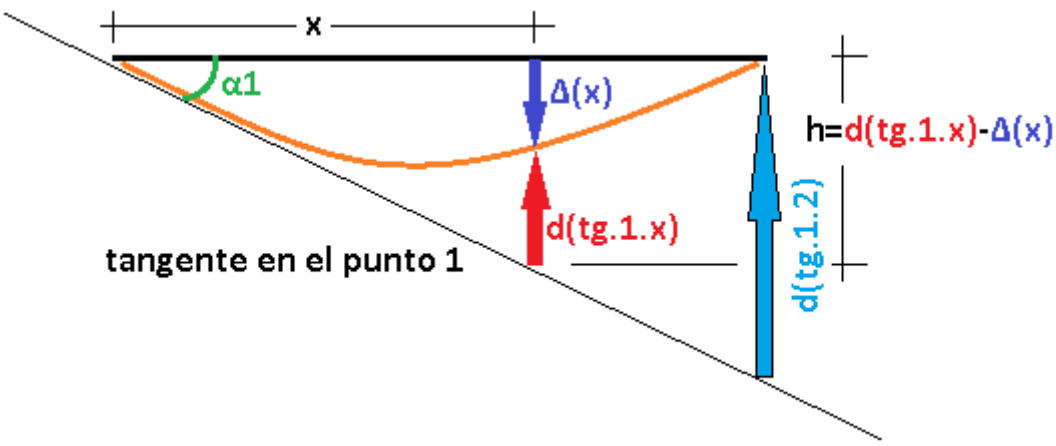
#10: $$cg(x) := \frac{x\cdot(x - 3\cdot L)}{3\cdot(x - 2\cdot L)}$$

Cálculo del momento flector, MF1(x)[0.a] y MF2(x)[a,L]:

#11: $$\begin{bmatrix} MF1(x) := Y1\cdot x - WT(x)\cdot cg(x) \\ MF2(x) := Y1\cdot x - WT(x)\cdot cg(x) - P\cdot(x - a) \end{bmatrix}$$

#12: $$\begin{bmatrix} MF1(x) := \frac{W\cdot x\cdot(2\cdot L^2 - 3\cdot L\cdot x + x^2)}{6\cdot L} + \frac{P\cdot x\cdot(L - a)}{L} \\ MF2(x) := \frac{P\cdot a\cdot(L - x)}{L} + \frac{W\cdot x\cdot(2\cdot L^2 - 3\cdot L\cdot x + x^2)}{6\cdot L} \end{bmatrix}$$

Cálculo de la deflexión en cualquier punto x, Δ1(x)[0,a] y Δ2(x)[a,L]:



#13: $$dtg12 = \frac{1}{E\cdot I}\cdot\left(\int_0^a MF1(x)\cdot(L - x)\ dx + \int_a^L MF2(x)\cdot(L - x)\ dx\right)$$

#14: $$dtg12 := \frac{P\cdot a\cdot(2\cdot L^2 - 3\cdot L\cdot a + a^2)}{6\cdot E\cdot I} + \frac{L^4\cdot W}{45\cdot E\cdot I}$$

#15: $$d1\_tg1(X) := \frac{1}{E\cdot I}\cdot\int_0^X MF1(x)\cdot(X - x)\ dx$$

#16: $$d1\_tg1(X) := \frac{P\cdot X^3\cdot(L - a)}{6\cdot E\cdot I\cdot L} + \frac{W\cdot X^3\cdot(20\cdot L^2 - 15\cdot L\cdot X + 3\cdot X^2)}{360\cdot E\cdot I\cdot L}$$

#17: $$d2\_tg1(X) := \frac{1}{E\cdot I}\cdot\left(\int_0^a MF1(x)\cdot(X - x)\ dx + \int_a^X MF2(x)\cdot(X - x)\ dx\right)$$

#18: $$d2\_tg1(X) := \frac{W\cdot X^3\cdot(20\cdot L^2 - 15\cdot L\cdot X + 3\cdot X^2)}{360\cdot E\cdot I\cdot L} + \frac{P\cdot a\cdot(L\cdot(3\cdot X^2 - 3\cdot X\cdot a + a^2) - X^3)}{6\cdot E\cdot I\cdot L}$$

#19: $$\left[\frac{dtg12}{L} = \frac{h}{x},\ d1\_tg1(x) :=,\ d2\_tg1(x) :=,\ \Delta1(X) :=,\ \Delta2(X) :=\right]$$

#20: $$h = dtg12\cdot\frac{x}{L}$$

#21:
$$\begin{bmatrix} d1\_tg1(X) - \Delta\_1 = dtg12\cdot\frac{x}{L} \\ d2\_tg1(X) - \Delta\_2 = dtg12\cdot\frac{x}{L} \end{bmatrix}$$

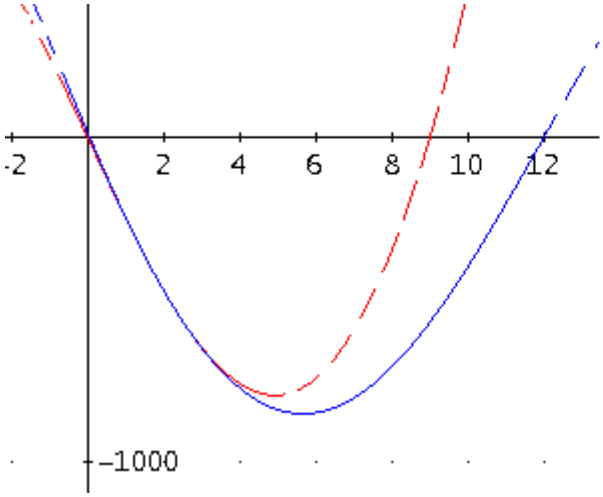
#22:
$$\left[\Delta\_1 = \frac{L\cdot d1\_tg1(X) - dtg12\cdot x}{L} \wedge \Delta\_2 = \frac{L\cdot d2\_tg1(X) - dtg12\cdot x}{L}\right]$$

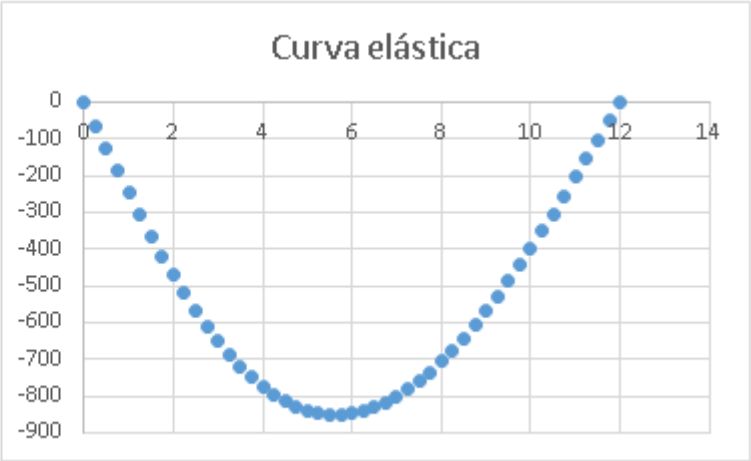
#23:
$$\begin{bmatrix} \Delta1(x) := \frac{L\cdot d1\_tg1(x) - dtg12\cdot x}{L} \\ \Delta2(x) := \frac{L\cdot d2\_tg1(x) - dtg12\cdot x}{L} \end{bmatrix}$$

#24:
$$\begin{bmatrix} \Delta1(x) := -\frac{W\cdot x\cdot(8\cdot L^4 - 20\cdot L^2\cdot x^2 + 15\cdot L\cdot x^3 - 3\cdot x^4)}{360\cdot E\cdot I\cdot L} \\ \Delta2(x) := \frac{W\cdot x^5}{120\cdot E\cdot I\cdot L} - \frac{W\cdot x^4}{24\cdot E\cdot I} + \frac{L\cdot W\cdot x^3}{18\cdot E\cdot I} - \frac{P\cdot a\cdot x^3}{6\cdot E\cdot I\cdot L} + \\ - \frac{P\cdot x\cdot(2\cdot L^2\cdot a - L\cdot(3\cdot a^2 + x^2) + a\cdot(a^2 + x^2))}{6\cdot E\cdot I\cdot L} \\ \frac{P\cdot a\cdot x^2}{2\cdot E\cdot I} - \frac{L^3\cdot W\cdot x}{45\cdot E\cdot I} - \frac{P\cdot a\cdot x\cdot(2\cdot L^2 + a^2)}{6\cdot E\cdot I\cdot L} + \frac{P\cdot a^3}{6\cdot E\cdot I} \end{bmatrix}$$

Usando datos para P,W,a,L,E,I:
SUBST([#24],[W,P,a,L,E,I],[5,10,2,12,1,1])

#25:
$$\begin{bmatrix} \Delta1(x) := \frac{x\cdot(x^4 - 60\cdot x^3 + 1360\cdot x^2 - 72896)}{288} \\ \Delta2(x) := \frac{(x - 12)\cdot(x^4 - 48\cdot x^3 + 304\cdot x^2 + 6528\cdot x - 320)}{288} \end{bmatrix}$$


-2
2
4
6
8
10
12
-1000


Curva elástica
0
-100
-200
-300
-400
-500
-600
-700
-800
-900
0
2
4
6
8
10
12
14