ESPECIALIZACIÓN EN ESTRUCTURAS
ANÁLISIS ESTRUCTURAL AVANZADO

ELEMENTOS UNIDIMENSIONALES
CASOS ESPECIALES

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ELEMENTOS UNIDIMENSIONALES
CASOS ESPECIALES

1. Igualación de GDL.
2. Condensación de GDL.
3. Subestructuración.
4. Articulaciones.
5. Nudos rígidos.

1. IGUALACIÓN DE GRADOS DE LIBERTAD

$$[A]\{U\} = \{0\}$$

$$\begin{bmatrix} 3 & 4 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 3 & -4 & 0 \end{bmatrix} \cdot \begin{bmatrix} ux1 \\ uy1 \\ \theta x1 \\ ux2 \\ uy2 \\ \theta x2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$[A_d \quad A_i] \begin{Bmatrix} U_i \\ U_d \end{Bmatrix} = \{0\}$$

$$\left[\begin{bmatrix} 3 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 4 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 3 & -4 \end{bmatrix}\right] \cdot \begin{bmatrix} \begin{bmatrix} ux1 \\ \theta x1 \\ \theta x2 \end{bmatrix} \\ \begin{bmatrix} uy1 \\ ux2 \\ uy2 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\{U_d\} = -[A_d]^{-1}[A_i]\{U_i\} = [R_o]\{U_i\}$$

$$\{U\} = \begin{Bmatrix} U_i \\ U_d \end{Bmatrix} = \quad = [R]\{U_i\}$$

$$Ro := \begin{bmatrix} -\frac{3}{4} & 0 & 0 \\ 1 & 0 & 0 \\ \frac{3}{4} & 0 & 0 \end{bmatrix}$$

$$R := \begin{bmatrix} 1 & 0 & 0 \\ -\frac{3}{4} & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ \frac{3}{4} & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

1. IGUALACIÓN DE GRADOS DE LIBERTAD

$$[F] = \{K\}\{U\}$$

$$F = \begin{bmatrix} \frac{2\cdot(17\cdot\psi+96)}{25} & \frac{12\cdot(\psi-12)}{25} & \frac{24}{5} & -\psi & 0 & 0 \\ \frac{12\cdot(\psi-12)}{25} & \frac{8\cdot(2\cdot\psi+51)}{25} & \frac{12}{5} & 0 & -12 & 6 \\ \frac{24}{5} & \frac{12}{5} & 8 & 0 & -6 & 2 \\ -\psi & 0 & 0 & \frac{2\cdot(17\cdot\psi+96)}{25} & \frac{12\cdot(12-\psi)}{25} & \frac{24}{5} \\ 0 & -12 & -6 & \frac{12\cdot(12-\psi)}{25} & \frac{8\cdot(2\cdot\psi+51)}{25} & -\frac{12}{5} \\ 0 & 6 & 2 & \frac{24}{5} & -\frac{12}{5} & 8 \end{bmatrix} \cdot \begin{bmatrix} ux1 \\ \theta x1 \\ \theta x2 \\ uy1 \\ ux2 \\ uy2 \end{bmatrix}$$

$$\{F_i\} = [R]^T\{F\} = [R]^T(\{K\}\{U\}) = [R]^T\left(\{K\}\{[R]\{U_i\}\}\right) = ([R]^T\{K\}[R])\{U_i\} = [K_i]\{U_i\}$$

$$Ki :=$$

2. CONDENSACIÓN DE GDL

$$[F] = [K]\{U\}$$

$$\{U\} = \begin{Bmatrix} U_c \\ U_0 \end{Bmatrix}$$

$$[K] = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}$$

$$[F] = \begin{Bmatrix} F_c \\ 0 \end{Bmatrix} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{Bmatrix} U_c \\ U_0 \end{Bmatrix}$$

$$[F_C] = [K_{11}]\{U_c\} + [K_{12}]\{U_o\}$$

$$[0] = [K_{21}]\{U_c\} + [K_{22}]\{U_o\}$$

$$\{U_o\} = -[K_{22}]^{-1}[K_{21}]\{U_c\}$$

$$[F_C] = ([K_{11}] - [K_{12}][K_{22}]^{-1}[K_{21}])\{U_c\}$$

$$[\boldsymbol{K_C}] = ([K_{11}] - [K_{12}][K_{22}]^{-1}[K_{21}])$$

$$[F_C] = [\boldsymbol{K_C}]\{U_c\}$$

$$\{U_c\} = [K_C]^{-1}[F_C]$$

$$\begin{bmatrix} Fx1 \\ M1=0 \\ M2=0 \end{bmatrix} = \frac{1}{25} \cdot \begin{bmatrix} 1612.5 & -37.5 & -37.5 \\ -37.5 & 200 & 50 \\ -37.5 & 50 & 200 \end{bmatrix} \cdot \begin{bmatrix} ux1 \\ \theta z1 \\ \theta z2 \end{bmatrix}$$

$[K_C] = ?$

$\{U_c\} = ?$

$\{U_0\} = ?$

$[F_C] = ?$

3. SUBESTRUCTURACIÓN

$$\{R\} + \{P\} = [K]\{D\} + \{EMP\}$$

$$\begin{Bmatrix} R_a \\ R_b \end{Bmatrix} + \begin{Bmatrix} P_a \\ P_b \end{Bmatrix} = \begin{bmatrix} K_{aa} & K_{ab} \\ K_{ba} & K_{bb} \end{bmatrix} \begin{Bmatrix} D_a \\ D_b \end{Bmatrix} + \begin{Bmatrix} EMP_a \\ EMP_b \end{Bmatrix}$$

$$\begin{Bmatrix} R_a = \{0\} \\ R_b \end{Bmatrix} + \begin{Bmatrix} P_a \\ P_b \end{Bmatrix} = \begin{bmatrix} K_{aa} & K_{ab} \\ K_{ba} & K_{bb} \end{bmatrix} \begin{Bmatrix} D_a \\ D_b = \{0\} \end{Bmatrix} + \begin{Bmatrix} EMP_a \\ EMP_b \end{Bmatrix}$$

3. SUBESTRUCTURACIÓN

$$\{R\} + \{P\} = [K]\{D\} + \{EMP\}$$

$$\begin{Bmatrix} \{0\} \\ R_b \end{Bmatrix} + \begin{Bmatrix} P_a \\ P_b \end{Bmatrix} = \begin{bmatrix} K_{aa} \\ K_{ba} \end{bmatrix} \{D_a\} + \begin{Bmatrix} EMP_a \\ EMP_b \end{Bmatrix}$$

$$\{0\} + \{P_a\} = [K_{aa}]\{D_a\} + \{EMP_a\}$$

$$\{R_b\} + \{P_b\} = [K_{ba}]\{D_a\} + \{EMP_b\}$$

3. SUBESTRUCTURACIÓN

$$\{P_a\} = [K_{aa}]\{D_a\} + \{EMP_a\}$$

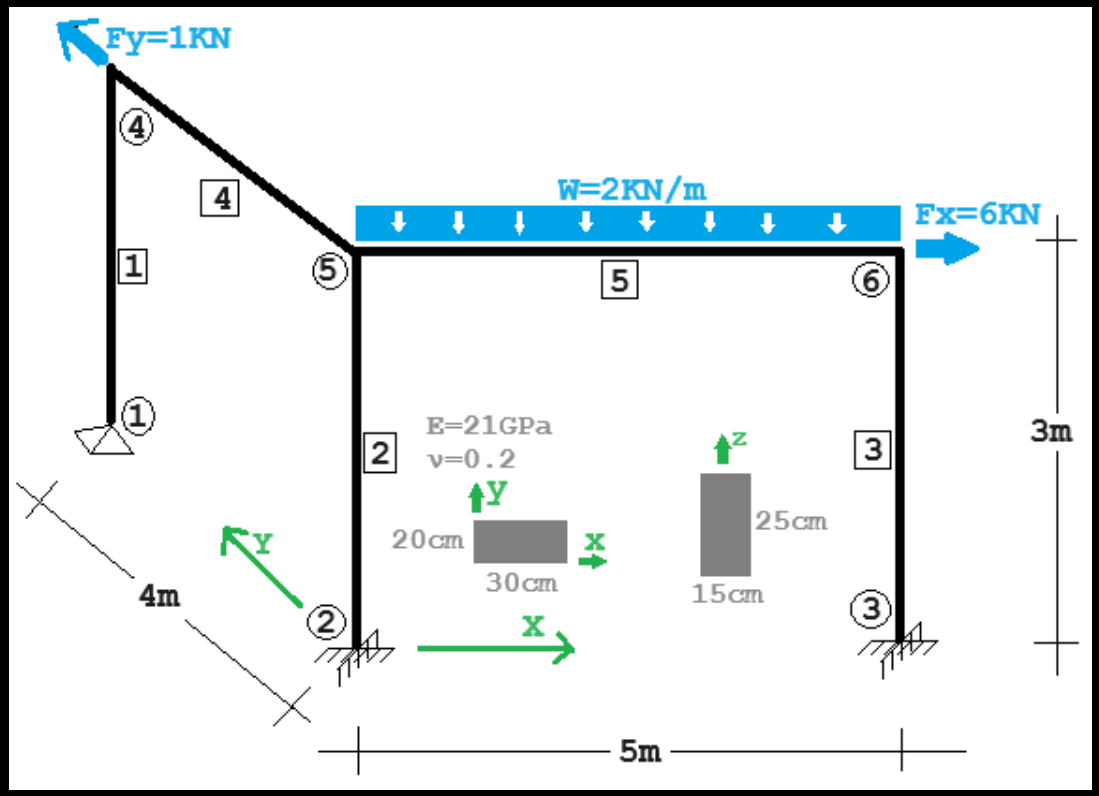
$$[K_{aa}]\{D_a\} = \{P_a\} - \{EMP_a\}$$

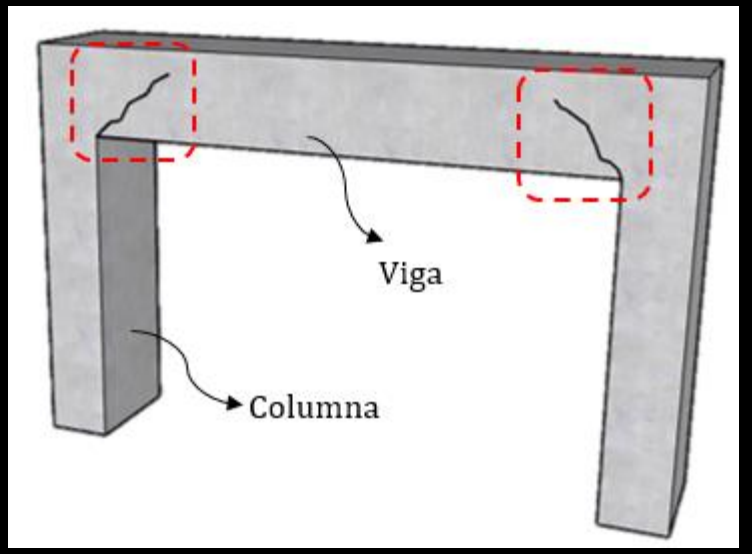
$$[K_{aa}]^{-1}[K_{aa}]\{D_a\} = [K_{aa}]^{-1}(\{P_a\} - \{EMP_a\})$$

$$\{D_a\} = [K_{aa}]^{-1}(\{P_a\} - \{EMP_a\})$$

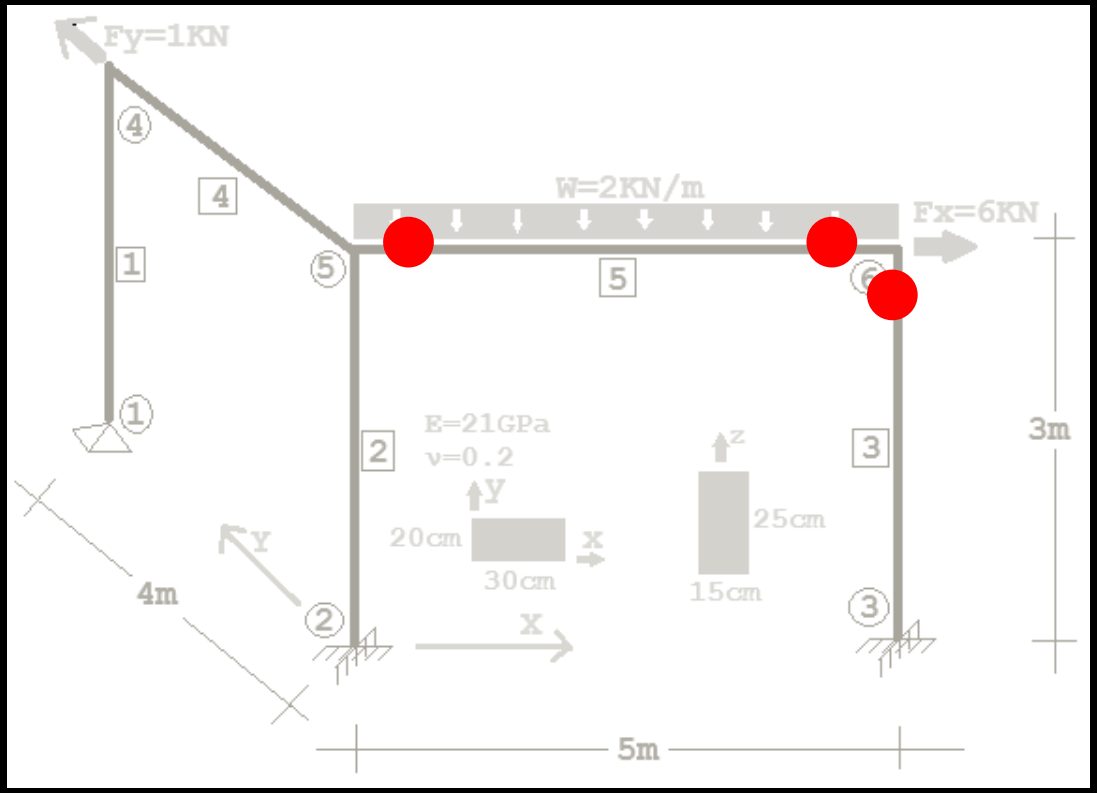
$$\{R_b\} = [K_{ba}]\{D_a\} + \{EMP_b\} - \{P_b\}$$

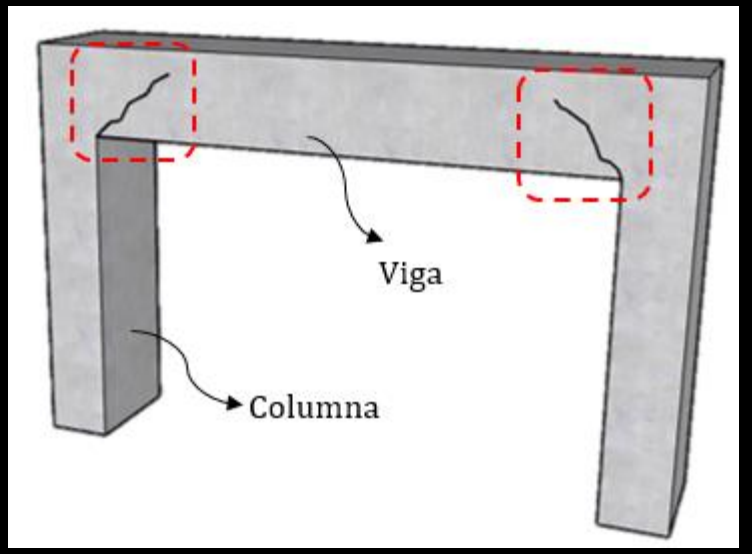
4. ARTICULACIONES





4. ARTICULACIONES





4. ARTICULACIONES

Condensar los gdl a articular:

$$[f'] = [k']\{u'\}$$

$$\{u'\} = \begin{Bmatrix} u'_c \\ u'_0 \end{Bmatrix}$$

$$[k'] = \begin{bmatrix} k'_{11} & k'_{12} \\ k'_{21} & k'_{22} \end{bmatrix}$$

$$[f'] = \begin{Bmatrix} F'_c \\ 0 \end{Bmatrix} = \begin{bmatrix} k'_{11} & k'_{12} \\ k'_{21} & k'_{22} \end{bmatrix} \begin{Bmatrix} u'_c \\ u'_0 \end{Bmatrix}$$

$$[f'_c] = [k'_{11}]\{u'_c\} + [k'_{12}]\{u'_o\}$$

$$[0] = [k'_{21}]\{u'_c\} + [k'_{22}]\{u'_o\}$$

$$\{u_o\} = -[k'_{22}]^{-1}[k'_{21}]\{u'_c\}$$

$$[f_C] = ([k'_{11}] - [k'_{12}][k'_{22}]^{-1}[k'_{21}])\{u'_c\}$$

$$[\boldsymbol{k'_c}] = ([k'_{11}] - [k'_{12}][k'_{22}]^{-1}[k'_{21}])$$

$$[f'_c] = [\boldsymbol{k'_c}]\{u'_c\}$$

$$[\boldsymbol{k"_c}] = adecuar[\boldsymbol{k'_c}]\ con\ filas\ y\ columnas\ de\ ceros\ en\ los\ gdl\ articulados$$

4. ARTICULACIONES

Condensar un solo (i) gdl a articular:

$$[f'] = [k']\{u'\}$$

$$\{u'\} = \begin{Bmatrix} u'_c \\ u'_0 \end{Bmatrix}$$

$$[k'] = \begin{bmatrix} k'_{11} & k'_{12} \\ k'_{21} & k'_{22} \end{bmatrix}$$

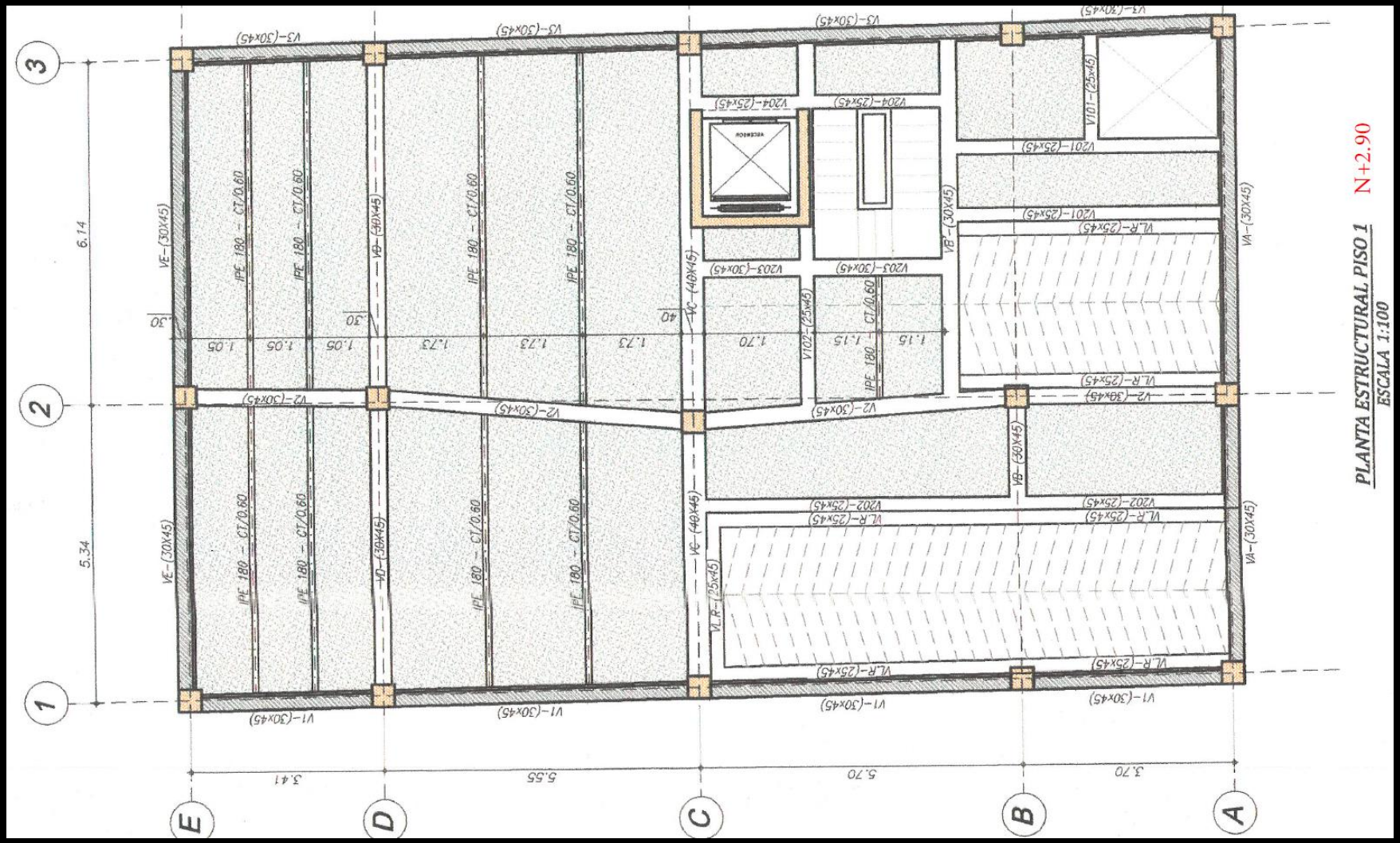
$$[\boldsymbol{k'_c}] = ([k'_{11}] - [k'_{12}][k'_{22}]^{-1}[k'_{21}])$$

$$[\boldsymbol{k'_c}] = \left([k'] - [k'_{columna\ i}][k'_{ii}]^{-1}[k'_{fila\ i}]\right)$$

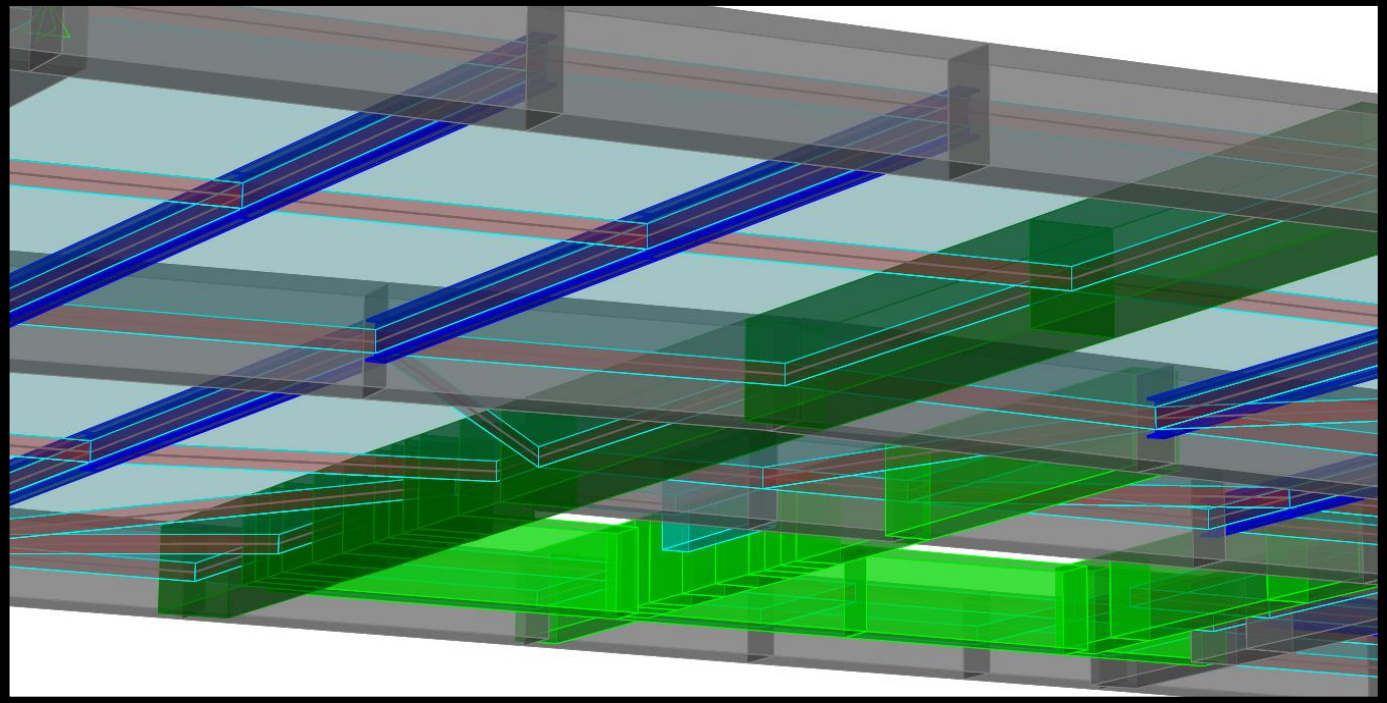
$$[f'_c] = [\boldsymbol{k'_c}]\{u'_c\}$$

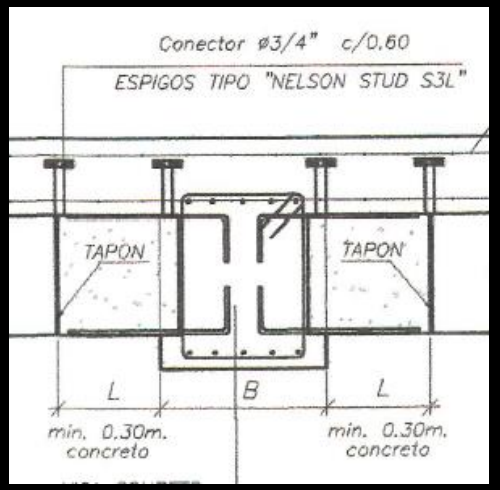
$\{emp'\}$: El vector de empotramiento debe adecuarse a las condiciones de apoyo articulado

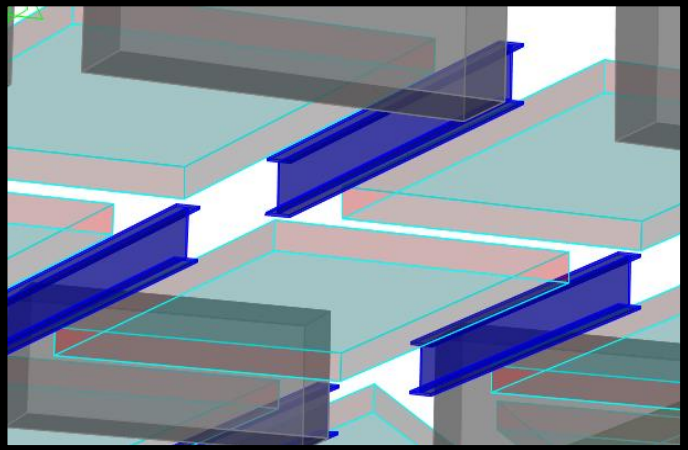
4. ARTICULACIONES

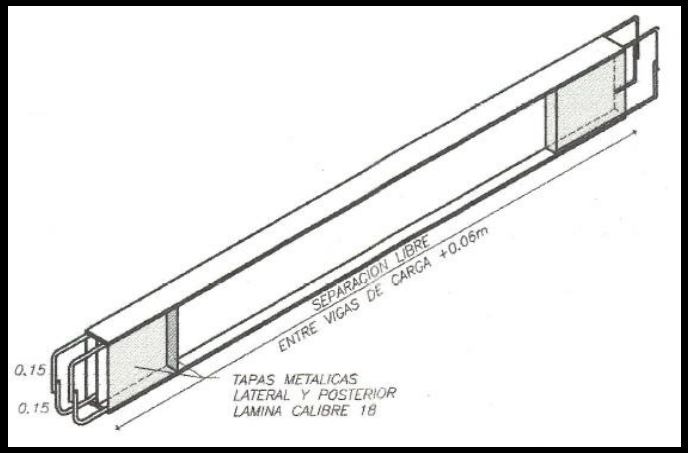


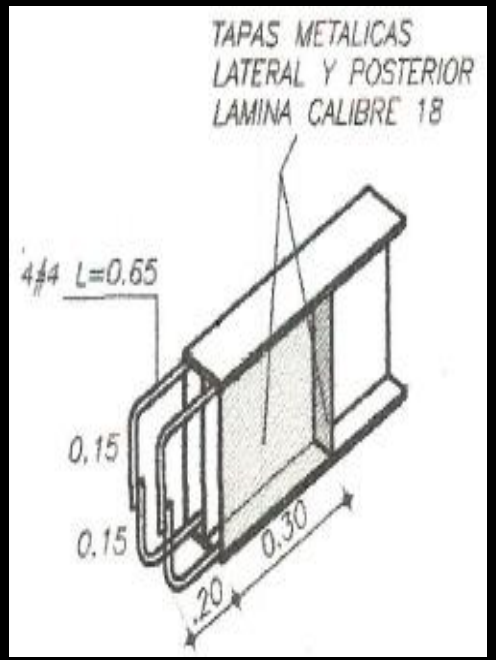
4. ARTICULACIONES











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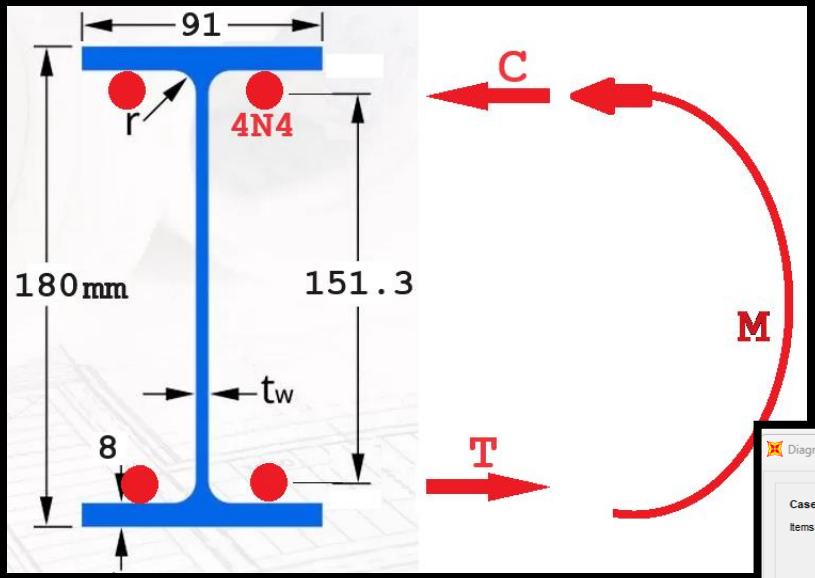
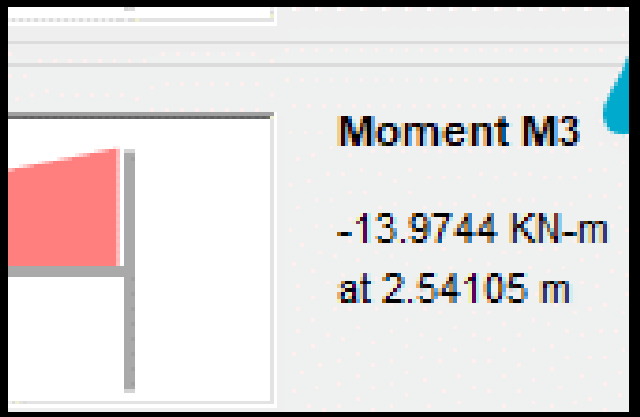
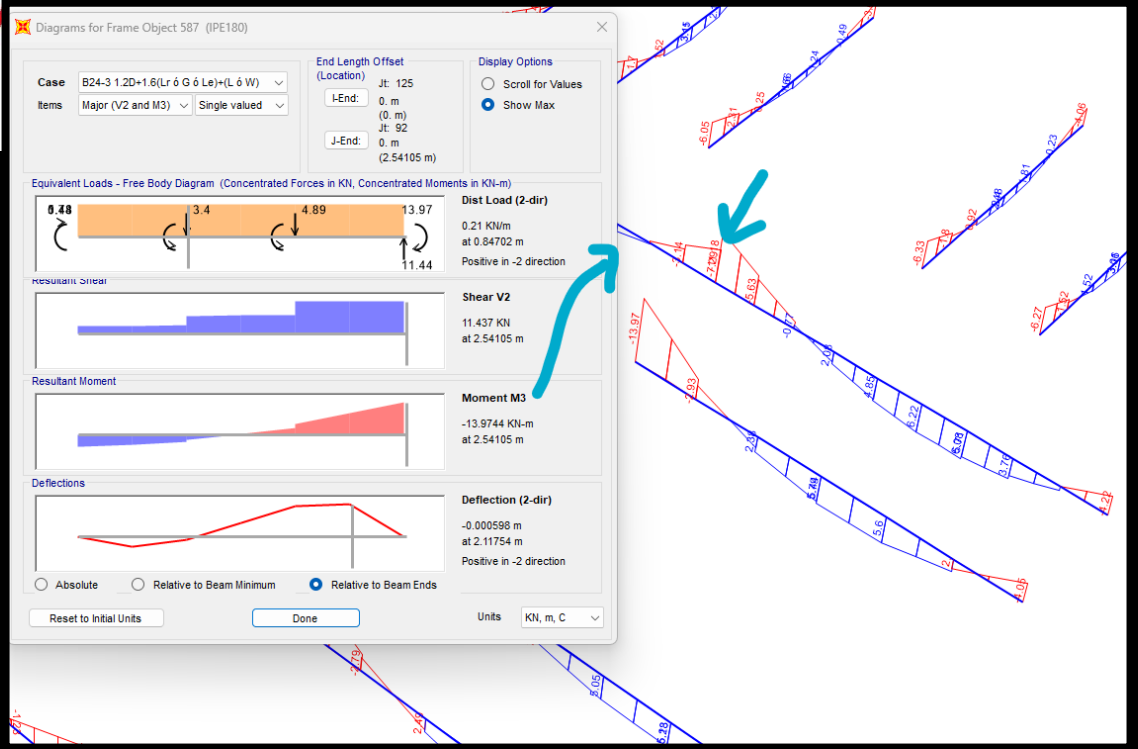




Para barras fy=420 Mpa:

Momento flector nominal máximo: 8.2 kN.m

Momento flector último máximo: 7.4 kN.m



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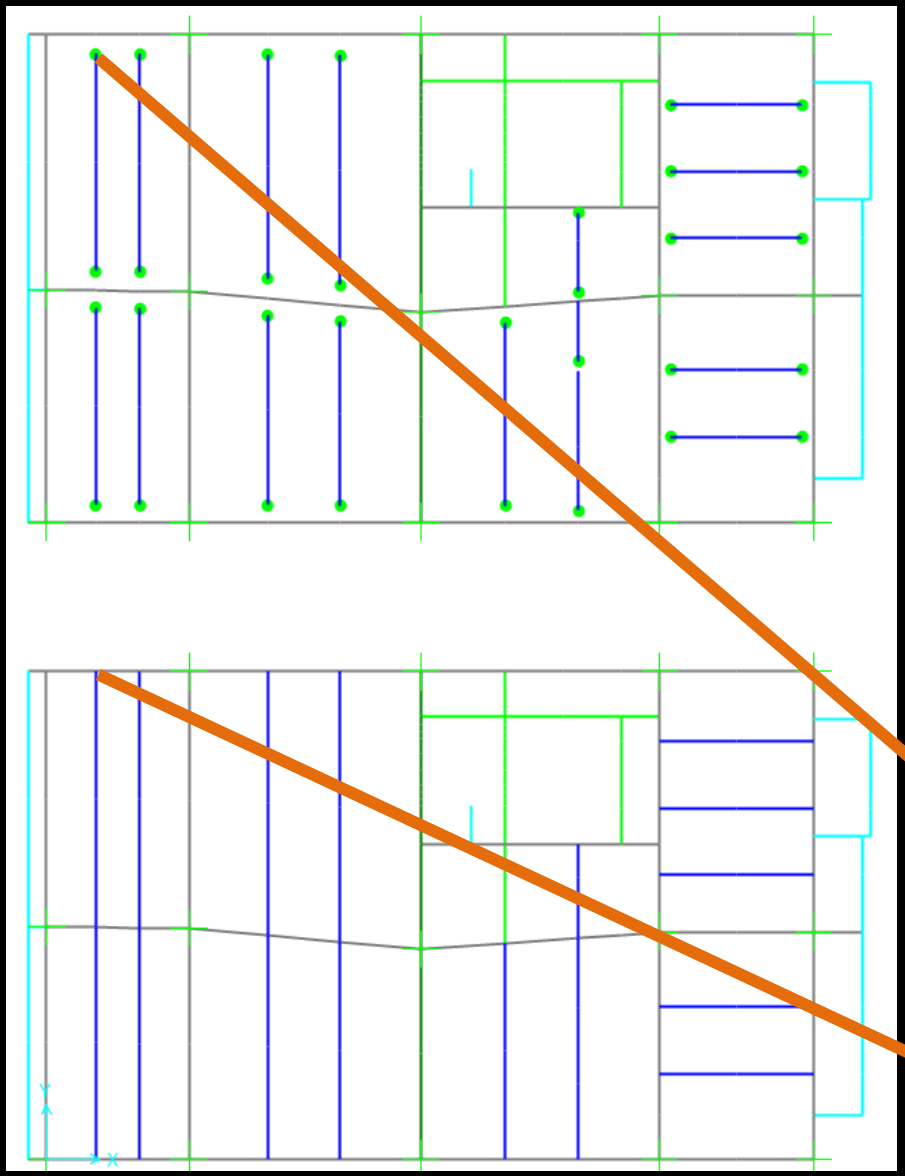

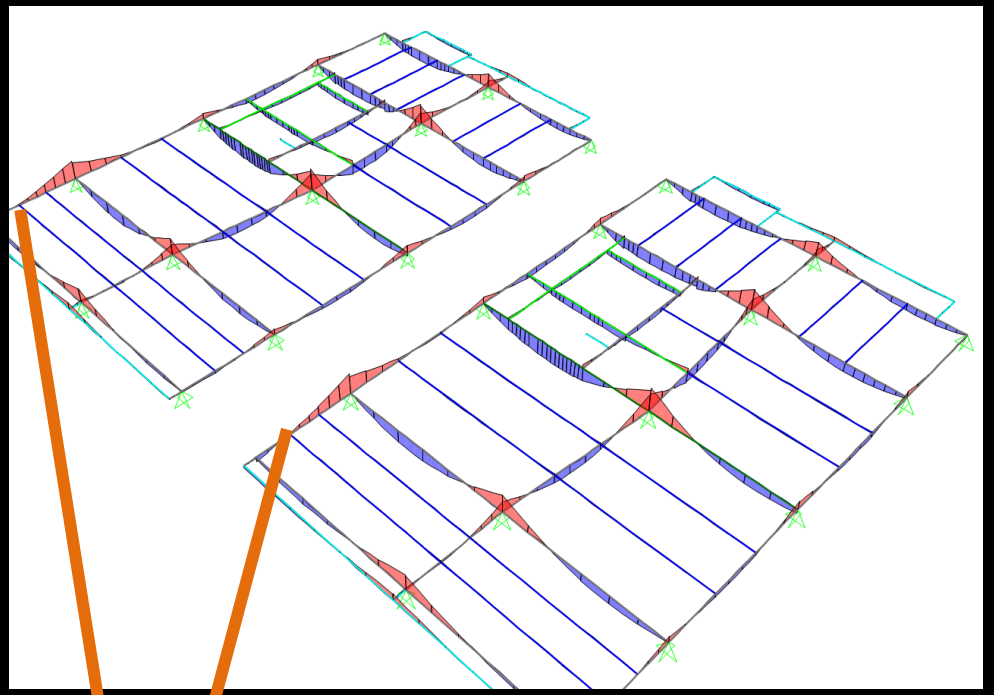

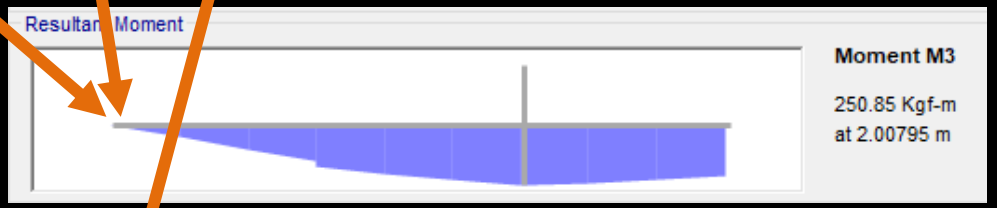

5. NUDOS RÍGIDOS

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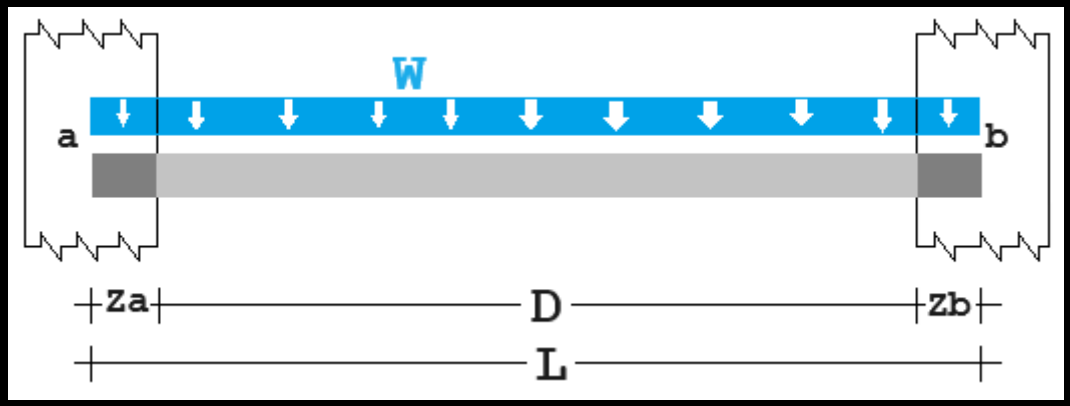
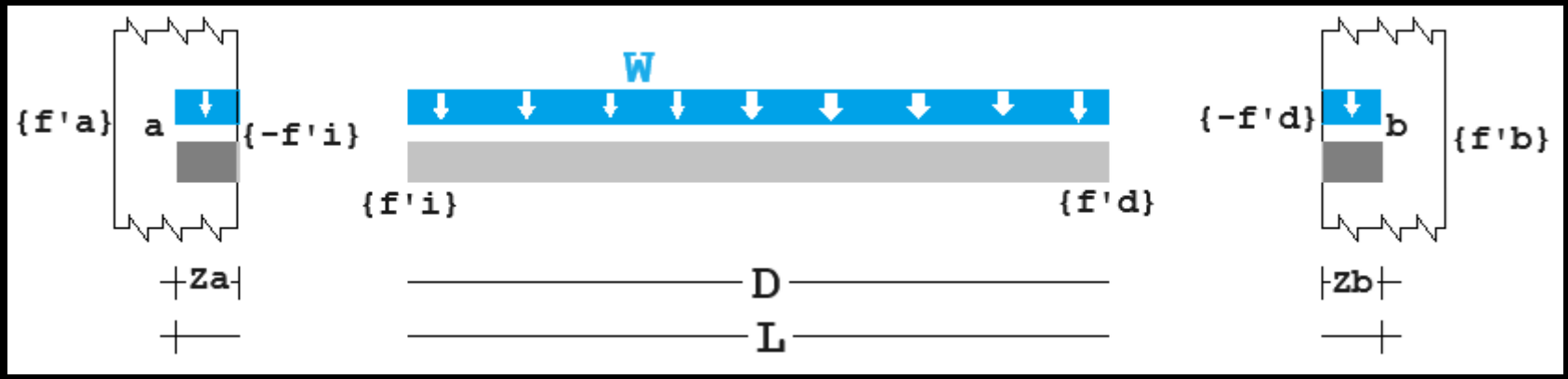


Diagramas de cuerpo libre:



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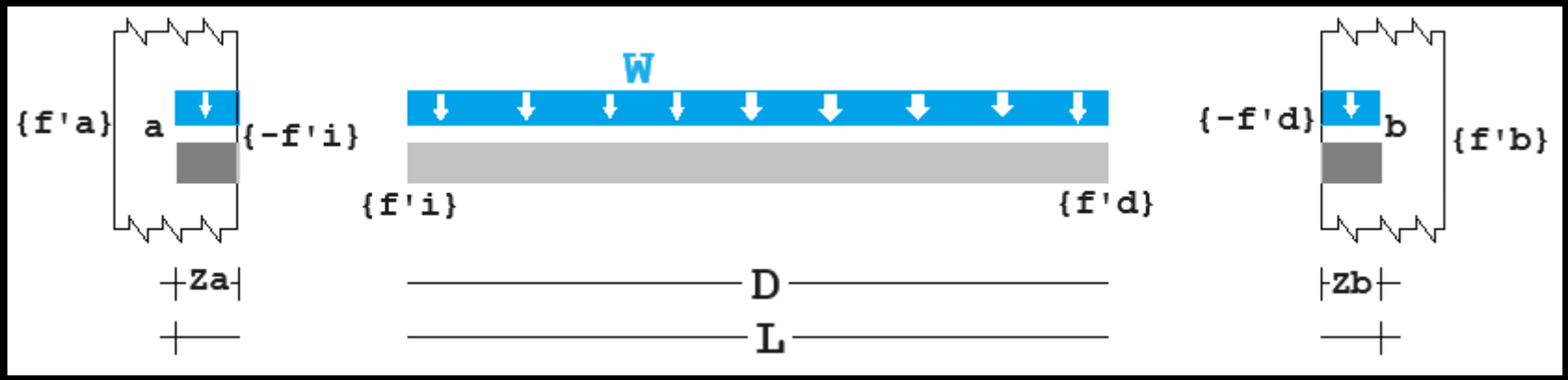
Diagramas de cuerpo libre:



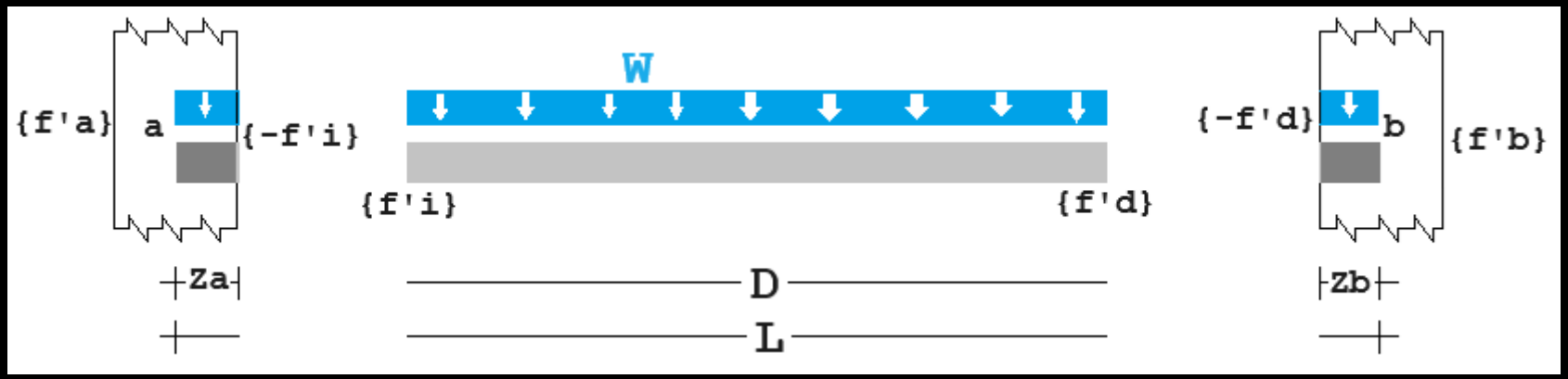
$$\{p'_L\} = \begin{Bmatrix} \{f'i\} \\ \{f'd\} \end{Bmatrix}$$

$$\{f'\} = [T]\{p'_L\}$$

$$\{f'\} = \begin{Bmatrix} x'_a \\ y'_a \\ mz'_a \\ x'_b \\ y'_b \\ mz'_b \end{Bmatrix} = \begin{Bmatrix} x'_i \\ y'_i \\ mz'_i + Za * y'_i \\ x'_d \\ y'_d \\ md'_d - Zb * y'_d \end{Bmatrix} = \begin{bmatrix} & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{bmatrix} \begin{Bmatrix} x'_i \\ y'_i \\ mz'_i \\ x'_d \\ y'_d \\ mz'_d \end{Bmatrix}$$

5. NUDOS RÍGIDOS

Diagramas de cuerpo libre:



$$\{p'_L\} = \begin{Bmatrix}\{f'i\}\\ \{f'd\}\end{Bmatrix}$$

$$\{p'_L\} = [k'_L]\{u'_L\}$$

$$\{f'\} = [T]\{p'_L\}$$

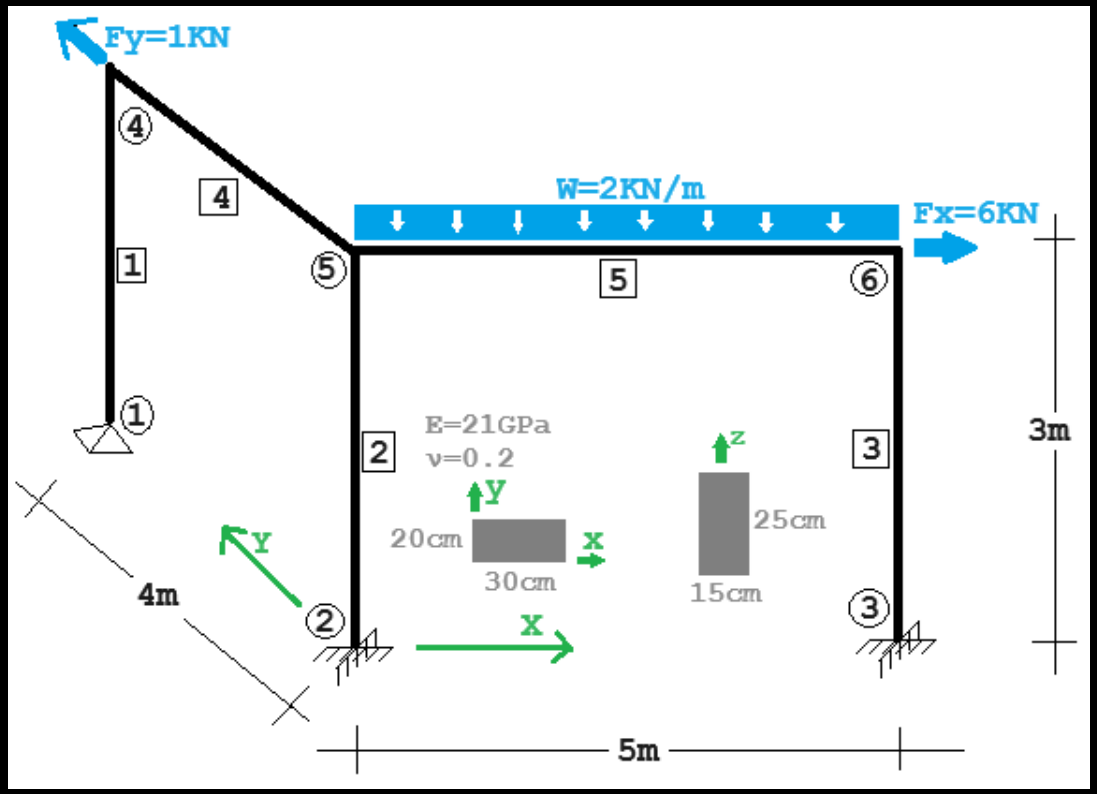
$$\{f'\} = [T][k'_L]\{u'_L\}$$

$$\{u'_L\} = [T]^T\{u'\}$$

$$\{f'\} = [\boldsymbol{T}][\boldsymbol{k'_L}][\boldsymbol{T}]^{\boldsymbol{T}}\{u'\}$$

$$\{f'\} = [\boldsymbol{k}]\{u'\}$$

5. NUDOS RÍGIDOS



Gracias

Créditos a:
https://openai.com/dall-e-2 - https://aminoapps.com/ - https://miprofe.com/ - https://www.youtube.com/@EASYCTE -

Michel Bolaños Guerrero, Ing. C., Esp., Mag.

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