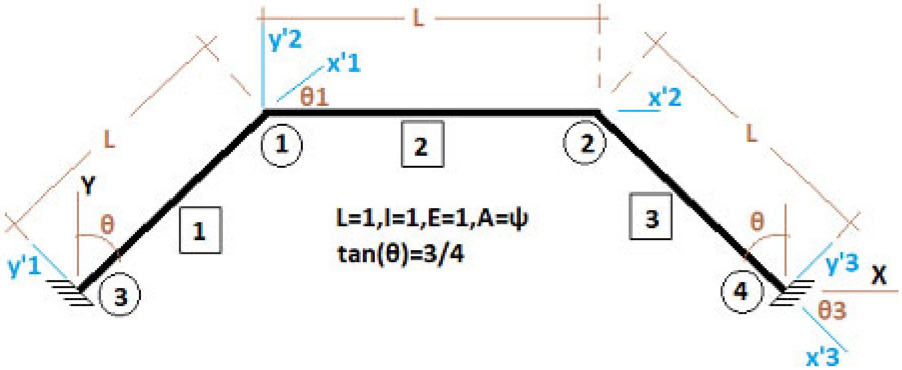
Desarrollo matricial del ejemplo 9-1 de García, L. E., Dinámica Estructural Aplicada al Diseño Sísmico, 1998



#1: [CaseMode:=Sensitive,InputMode:=Word,DisplayFormat:=Compressed]

#2: $\left[L:=1, I:=1, E:=1, A:=\psi, \theta:=ATAN\left(\frac{3}{4}\right), \theta1:=ATAN\left(\frac{4}{3}\right), \theta2:=0, \theta3:=ATAN\left(\frac{4}{3}\right)\right]$

Matriz de rigidez local en coordenadas locales, igual en los tres elementos:

#3: $$k:=\begin{bmatrix} \frac{A\cdot E}{L} & 0 & 0 & -\frac{A\cdot E}{L} & 0 & 0 \\ 0 & \frac{12\cdot E\cdot I}{L^3} & \frac{6\cdot E\cdot I}{L^2} & 0 & -\frac{12\cdot E\cdot I}{L^3} & \frac{6\cdot E\cdot I}{L^2} \\ 0 & \frac{6\cdot E\cdot I}{L^2} & \frac{4\cdot E\cdot I}{L} & 0 & -\frac{6\cdot E\cdot I}{L^2} & \frac{2\cdot E\cdot I}{L} \\ -\frac{A\cdot E}{L} & 0 & 0 & \frac{A\cdot E}{L} & 0 & 0 \\ 0 & -\frac{12\cdot E\cdot I}{L^3} & -\frac{6\cdot E\cdot I}{L^2} & 0 & \frac{12\cdot E\cdot I}{L^3} & -\frac{6\cdot E\cdot I}{L^2} \\ 0 & \frac{6\cdot E\cdot I}{L^2} & \frac{2\cdot E\cdot I}{L} & 0 & -\frac{6\cdot E\cdot I}{L^2} & \frac{4\cdot E\cdot I}{L} \end{bmatrix}$$

#4: $$k:=\begin{bmatrix} \psi & 0 & 0 & -\psi & 0 & 0 \\ 0 & 12 & 6 & 0 & -12 & 6 \\ 0 & 6 & 4 & 0 & -6 & 2 \\ -\psi & 0 & 0 & \psi & 0 & 0 \\ 0 & -12 & -6 & 0 & 12 & -6 \\ 0 & 6 & 2 & 0 & -6 & 4 \end{bmatrix}$$

Matriz de transformación general de coordenadas locales a coordenadas globales:

To=COS[(x x') (x y')|
        (y x') (y y')|]
v=To*v'

Matrices de transformación 2D básicas:

#5: $$To2D:=COS\begin{bmatrix} \theta x\_xp & \theta x\_yp \\ \theta y\_xp & \theta y\_yp \end{bmatrix}$$

#6: $$To2D1:=SUBST\left(COS\begin{bmatrix} \theta x\_xp & \theta x\_yp \\ \theta y\_xp & \theta y\_yp \end{bmatrix},[\theta x\_xp,\theta x\_yp,\theta y\_xp,\theta y\_yp],\left[\theta 1,\frac{\pi}{2}+\theta 1,\frac{\pi}{2}-\theta 1,\theta 1\right]\right)$$

#7: $$To2D1:=\begin{bmatrix} 0.6 & -0.8 \\ 0.8 & 0.6 \end{bmatrix}$$

#8: To2D2:=IDENTITY_MATRIX(2)

#9: $$To2D2:=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

#10: $$To2D3:=SUBST\left(COS\begin{bmatrix} \theta x\_xp & \theta x\_yp \\ \theta y\_xp & \theta y\_yp \end{bmatrix},[\theta x\_xp,\theta x\_yp,\theta y\_xp,\theta y\_yp],\left[\theta 3,\frac{\pi}{2}-\theta 3,\frac{\pi}{2}+\theta 3,\theta 3\right]\right)$$

#11: $$To2D3:=\begin{bmatrix} 0.6 & 0.8 \\ -0.8 & 0.6 \end{bmatrix}$$

#12: $$To3:=T3_{[1,2]}\downarrow\downarrow[1,2]$$

#13: $$To3:=\begin{bmatrix} 0.6 & 0.8 \\ 0.8 & 0.6 \end{bmatrix}$$

#14: $$To:=COS\begin{bmatrix} θx\_xp & θx\_yp & \frac{\pi}{2} \\ θy\_xp & θy\_yp & \frac{\pi}{2} \\ \frac{\pi}{2} & \frac{\pi}{2} & 0 \end{bmatrix}$$

#15: $$Oo:=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

#16: $$T=\begin{bmatrix} To & Oo \\ Oo & To \end{bmatrix}$$

#17: $$T=\begin{bmatrix} COS(θx\_xp) & COS(θx\_yp) & 0 & 0 & 0 & 0 \\ COS(θy\_xp) & COS(θy\_yp) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & COS(θx\_xp) & COS(θx\_yp) & 0 \\ 0 & 0 & 0 & COS(θy\_xp) & COS(θy\_yp) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Matrices de transformación de cada elemento:

#18: $$T1:=\begin{bmatrix} COS(θ1) & COS\left(\frac{\pi}{2}+θ1\right) & 0 & 0 & 0 & 0 \\ COS(θ) & COS(θ1) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & COS(θ1) & COS\left(\frac{\pi}{2}+θ1\right) & 0 \\ 0 & 0 & 0 & COS(θ) & COS(θ1) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

#19:
$$T1:=\begin{bmatrix} 0.6 & -0.8 & 0 & 0 & 0 & 0 \\ 0.8 & 0.6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.6 & -0.8 & 0 \\ 0 & 0 & 0 & 0.8 & 0.6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

#20:
$$T2:=\begin{bmatrix} COS(\theta2) & COS\left(\frac{\pi}{2}\right) & 0 & 0 & 0 & 0 \\ COS\left(\frac{\pi}{2}\right) & COS(0) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & COS(\theta2) & COS\left(\frac{\pi}{2}\right) & 0 \\ 0 & 0 & 0 & COS\left(\frac{\pi}{2}\right) & COS(0) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

#21:
$$T2:=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

#22: $$T3:=SUBST\left(\begin{bmatrix} COS(\theta x\_xp) & COS(\theta x\_yp) & 0 & 0 & 0 & 0 \\ COS(\theta y\_xp) & COS(\theta y\_yp) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & COS(\theta x\_xp) & COS(\theta x\_yp) & 0 \\ 0 & 0 & 0 & COS(\theta y\_xp) & COS(\theta y\_yp) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, [\theta x\_xp, \theta x\_yp, \theta y\_xp, \theta y\_yp], \left[\theta 3, \frac{\pi}{2} - \theta 3, \frac{\pi}{2} + \theta 3, \theta 3\right]\right)$$

#23: $$T3:=\begin{bmatrix} 0.6 & 0.8 & 0 & 0 & 0 & 0 \\ -0.8 & 0.6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.6 & 0.8 & 0 \\ 0 & 0 & 0 & -0.8 & 0.6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Matrices de rigidez en coordenadas globales:

#24: $k1:=T1\cdot k\cdot T1'$

#25: $$k1:=\frac{1}{25}\cdot\begin{bmatrix} 9\cdot\psi+192 & 12\cdot\psi-144 & -120 & -9\cdot\psi-192 & 144-12\cdot\psi & -120 \\ 12\cdot\psi-144 & 16\cdot\psi+108 & 90 & 144-12\cdot\psi & -16\cdot\psi-108 & 90 \\ -120 & 90 & 100 & 120 & -90 & 50 \\ -9\cdot\psi-192 & 144-12\cdot\psi & 120 & 9\cdot\psi+192 & 12\cdot\psi-144 & 120 \\ 144-12\cdot\psi & -16\cdot\psi-108 & -90 & 12\cdot\psi-144 & 16\cdot\psi+108 & -90 \\ -120 & 90 & 50 & 120 & -90 & 100 \end{bmatrix}$$

k1 eliminando los gdl restringidos:

#26: $k1_{[4,5,6]} \downarrow\downarrow [4,5,6]$

#27:

$$\begin{bmatrix} \frac{9\cdot\psi}{25}+\frac{192}{25} & -\frac{12}{25}\cdot(12-\psi) & \frac{24}{5} \\ -\frac{12}{25}\cdot(12-\psi) & \frac{16\cdot\psi}{25}+\frac{108}{25} & -\frac{18}{5} \\ \frac{24}{5} & -\frac{18}{5} & 4 \end{bmatrix}$$

#28: k2:=T2·k·T2'

#29:

$$k2:=\begin{bmatrix} \psi & 0 & 0 & -\psi & 0 & 0 \\ 0 & 12 & 6 & 0 & -12 & 6 \\ 0 & 6 & 4 & 0 & -6 & 2 \\ -\psi & 0 & 0 & \psi & 0 & 0 \\ 0 & -12 & -6 & 0 & 12 & -6 \\ 0 & 6 & 2 & 0 & -6 & 4 \end{bmatrix}$$

#30: k3:=T3·k·T3'

#31:

$$k3:=\begin{bmatrix} \frac{9\cdot\psi}{25}+\frac{192}{25} & \frac{144}{25}-\frac{12\cdot\psi}{25} & \frac{24}{5} & -\frac{9\cdot\psi}{25}-\frac{192}{25} & \frac{12\cdot\psi}{25}-\frac{144}{25} & \frac{24}{5} \\ \frac{144}{25}-\frac{12\cdot\psi}{25} & \frac{16\cdot\psi}{25}+\frac{108}{25} & \frac{18}{5} & \frac{12\cdot\psi}{25}-\frac{144}{25} & -\frac{16\cdot\psi}{25}-\frac{108}{25} & \frac{18}{5} \\ \frac{24}{5} & \frac{18}{5} & 4 & -\frac{24}{5} & -\frac{18}{5} & 2 \\ -\frac{9\cdot\psi}{25}-\frac{192}{25} & \frac{12\cdot\psi}{25}-\frac{144}{25} & -\frac{24}{5} & \frac{9\cdot\psi}{25}+\frac{192}{25} & \frac{144}{25}-\frac{12\cdot\psi}{25} & -\frac{24}{5} \\ \frac{12\cdot\psi}{25}-\frac{144}{25} & -\frac{16\cdot\psi}{25}-\frac{108}{25} & -\frac{18}{5} & \frac{144}{25}-\frac{12\cdot\psi}{25} & \frac{16\cdot\psi}{25}+\frac{108}{25} & -\frac{18}{5} \\ \frac{24}{5} & \frac{18}{5} & 2 & -\frac{24}{5} & -\frac{18}{5} & 4 \end{bmatrix}$$

k3 eliminando los gdl restringidos:

#32: $k3_{[1,2,3]} \downarrow\downarrow [1,2,3]$

#33:
$$\begin{bmatrix} \frac{9\cdot\psi}{25}+\frac{192}{25} & \frac{12}{25}\cdot(12-\psi) & \frac{24}{5} \\ \frac{12}{25}\cdot(12-\psi) & \frac{16\cdot\psi}{25}+\frac{108}{25} & \frac{18}{5} \\ \frac{24}{5} & \frac{18}{5} & 4 \end{bmatrix}$$

Ensamblaje de la matriz de rigidez global:

Se agrandan inicialmente cada matriz local en coordenadas globales y se ubican sus respectivos elementos

#34:
$$k1g := \begin{bmatrix} k1_{4,4} & k1_{4,5} & k1_{4,6} & 0 & 0 & 0 & k1_{4,1} & k1_{4,2} & k1_{4,3} & 0 & 0 & 0 \\ k1_{5,4} & k1_{5,5} & k1_{5,6} & 0 & 0 & 0 & k1_{5,1} & k1_{5,2} & k1_{5,3} & 0 & 0 & 0 \\ k1_{6,4} & k1_{6,5} & k1_{6,6} & 0 & 0 & 0 & k1_{6,1} & k1_{6,2} & k1_{6,3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ k1_{1,4} & k1_{1,5} & k1_{1,6} & 0 & 0 & 0 & k1_{1,1} & k1_{1,2} & k1_{1,3} & 0 & 0 & 0 \\ k1_{2,4} & k1_{2,5} & k1_{2,6} & 0 & 0 & 0 & k1_{2,1} & k1_{2,2} & k1_{1,3} & 0 & 0 & 0 \\ k1_{3,4} & k1_{3,5} & k1_{3,6} & 0 & 0 & 0 & k1_{3,1} & k1_{3,2} & k1_{3,3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

#35: $$k2g := \begin{bmatrix} k2_{1,1} & k2_{1,2} & k2_{1,3} & k2_{1,4} & k2_{1,5} & k2_{1,6} & 0 & 0 & 0 & 0 & 0 & 0 \\ k2_{2,1} & k2_{2,2} & k2_{2,3} & k2_{2,4} & k2_{2,5} & k2_{2,6} & 0 & 0 & 0 & 0 & 0 & 0 \\ k2_{3,1} & k2_{3,2} & k2_{3,3} & k2_{3,4} & k2_{3,5} & k2_{3,6} & 0 & 0 & 0 & 0 & 0 & 0 \\ k2_{4,1} & k2_{4,2} & k2_{4,3} & k2_{4,4} & k2_{4,5} & k2_{4,6} & 0 & 0 & 0 & 0 & 0 & 0 \\ k2_{5,1} & k2_{5,2} & k2_{5,3} & k2_{5,4} & k2_{5,5} & k2_{5,6} & 0 & 0 & 0 & 0 & 0 & 0 \\ k2_{6,1} & k2_{6,2} & k2_{6,3} & k2_{6,4} & k2_{6,5} & k2_{6,6} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\#36: \quad k3g := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k3_{1,1} & k3_{1,2} & k3_{1,3} & 0 & 0 & 0 & k3_{1,4} & k3_{1,5} & k3_{1,6} \\ 0 & 0 & 0 & k3_{2,1} & k3_{2,2} & k3_{2,3} & 0 & 0 & 0 & k3_{2,4} & k3_{2,5} & k3_{2,6} \\ 0 & 0 & 0 & k3_{3,1} & k3_{3,2} & k3_{3,3} & 0 & 0 & 0 & k3_{3,4} & k3_{3,5} & k3_{3,6} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k3_{4,1} & k3_{4,2} & k3_{4,3} & 0 & 0 & 0 & k3_{4,4} & k3_{4,5} & k3_{4,6} \\ 0 & 0 & 0 & k3_{5,1} & k3_{5,2} & k3_{5,3} & 0 & 0 & 0 & k3_{5,4} & k3_{5,5} & k3_{5,6} \\ 0 & 0 & 0 & k3_{6,1} & k3_{6,2} & k3_{6,3} & 0 & 0 & 0 & k3_{6,4} & k3_{6,5} & k3_{6,6} \end{bmatrix}$$

Se suman las matrices agrandadas factorizando el 1/25:

$$\#37: \quad K := \frac{1}{25} \cdot (25 \cdot (k1g + k2g + k3g))$$

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#38:

$$K:=\frac{1}{25}\cdot\begin{bmatrix}
34\cdot\psi+192 & 12\cdot\psi-144 & 120 & -25\cdot\psi & 0 & 0 & -9\cdot\psi-192 & 144-12\cdot\psi & 120 & 0 & 0 & 0 \\
12\cdot\psi-144 & 16\cdot\psi+408 & 60 & 0 & -300 & 150 & 144-12\cdot\psi & -16\cdot\psi-108 & -90 & 0 & 0 & 0 \\
120 & 60 & 200 & 0 & -150 & 50 & -120 & 90 & 50 & 0 & 0 & 0 \\
-25\cdot\psi & 0 & 0 & 34\cdot\psi+192 & 144-12\cdot\psi & 120 & 0 & 0 & 0 & -9\cdot\psi-192 & 12\cdot\psi-144 & 120 \\
0 & -300 & -150 & 144-12\cdot\psi & 16\cdot\psi+408 & -60 & 0 & 0 & 0 & 12\cdot\psi-144 & -16\cdot\psi-108 & 90 \\
0 & 150 & 50 & 120 & -60 & 200 & 0 & 0 & 0 & -120 & -90 & 50 \\
-9\cdot\psi-192 & 144-12\cdot\psi & -120 & 0 & 0 & 0 & 9\cdot\psi+192 & 12\cdot\psi-144 & -120 & 0 & 0 & 0 \\
144-12\cdot\psi & -16\cdot\psi-108 & 90 & 0 & 0 & 0 & 12\cdot\psi-144 & 16\cdot\psi+108 & -120 & 0 & 0 & 0 \\
120 & -90 & 50 & 0 & 0 & 0 & -120 & 90 & 100 & 0 & 0 & 0 \\
0 & 0 & 0 & -9\cdot\psi-192 & 12\cdot\psi-144 & -120 & 0 & 0 & 0 & 9\cdot\psi+192 & 144-12\cdot\psi & -120 \\
0 & 0 & 0 & 12\cdot\psi-144 & -16\cdot\psi-108 & -90 & 0 & 0 & 0 & 144-12\cdot\psi & 16\cdot\psi+108 & -90 \\
0 & 0 & 0 & 120 & 90 & 50 & 0 & 0 & 0 & -120 & -90 & 100
\end{bmatrix}$$

Aplicación de las restricciones

Se eliminan las filas y columnas de los grados de libertad restringidos y se factoriza 1/25:

#39: $Kr:=K_{[1,2,3,4,5,6]\downarrow\downarrow[1,2,3,4,5,6]}$

#40:
$$Kr:=\frac{1}{25}\cdot\begin{bmatrix} 34\cdot\psi+192 & 12\cdot\psi-144 & 120 & -25\cdot\psi & 0 & 0 \\ 12\cdot\psi-144 & 16\cdot\psi+408 & 60 & 0 & -300 & 150 \\ 120 & 60 & 200 & 0 & -150 & 50 \\ -25\cdot\psi & 0 & 0 & 34\cdot\psi+192 & 144-12\cdot\psi & 120 \\ 0 & -300 & -150 & 144-12\cdot\psi & 16\cdot\psi+408 & -60 \\ 0 & 150 & 50 & 120 & -60 & 200 \end{bmatrix}$$

A continuación se adopta que no habrá deformación axial de los elementos, es decir, los elementos son infinitamente rígidos

Deformaciones de los nudos basados en una rigidez infinita, osea ux'=0;

Desplazamiento del nudo 1 por deformación del elemento 1:

#41: $$\begin{bmatrix} ux1 \\ uy1 \end{bmatrix}=To2D1\cdot\begin{bmatrix} uxp1 \\ uyp1 \end{bmatrix}$$

#42: $$\begin{bmatrix} ux1 \\ uy1 \end{bmatrix}=\begin{bmatrix} 0.6 & -0.8 \\ 0.8 & 0.6 \end{bmatrix}\cdot\begin{bmatrix} uxp1 \\ uyp1 \end{bmatrix}$$

#43: $$\begin{bmatrix} ux1 \\ uy1 \end{bmatrix}=\begin{bmatrix} 0.6\cdot uxp1-0.8\cdot uyp1 \\ 0.8\cdot uxp1+0.6\cdot uyp1 \end{bmatrix}$$

ux'1=0:

#44: $$\begin{bmatrix} ux1 \\ uy1 \end{bmatrix}=\begin{bmatrix} 0.6\cdot 0-0.8\cdot uyp1 \\ 0.8\cdot 0+0.6\cdot uyp1 \end{bmatrix}$$

#45: $$uyp1=-\frac{5\cdot ux1}{4}\wedge uyp1=\frac{5\cdot uy1}{3}$$

#46: $$-\frac{5\cdot ux1}{4}=\frac{5\cdot uy1}{3}$$

#47: $$3\cdot ux1+4\cdot uy1=0$$

Deformación del elemento 2 basado en que los desplazamiento axiales son iguales en los nudos:

#48: $\begin{bmatrix} ux1 \\ uy1 \end{bmatrix} = To2D2 \cdot \begin{bmatrix} uxp1 \\ uyp1 \end{bmatrix}$

#49: $\begin{bmatrix} ux1 \\ uy1 \end{bmatrix} = \begin{bmatrix} uxp1 \\ uyp1 \end{bmatrix}$

#50: $\begin{bmatrix} ux2 \\ uy2 \end{bmatrix} = To2D2 \cdot \begin{bmatrix} uxp2 \\ uyp2 \end{bmatrix}$

#51: $\begin{bmatrix} ux2 \\ uy2 \end{bmatrix} = \begin{bmatrix} uxp2 \\ uyp2 \end{bmatrix}$

#52: $uxp1=uxp2=uxp$

#53: $\begin{bmatrix} ux1 \\ uy1 \end{bmatrix} = \begin{bmatrix} uxp \\ uyp1 \end{bmatrix}$

#54: $\begin{bmatrix} ux2 \\ uy2 \end{bmatrix} = \begin{bmatrix} uxp \\ uyp2 \end{bmatrix}$

#55: $ux1=uxp \wedge uy1=uyp1$

#56: $ux2=uxp \wedge uy2=uyp2$

#57: $ux1-ux2=0$

Desplazamiento del nudo 2 por deformación del elemento 3:

#58: $\begin{bmatrix} ux2 \\ uy2 \end{bmatrix} = To2D3 \cdot \begin{bmatrix} uxp2 \\ uyp2 \end{bmatrix}$

#59: $\begin{bmatrix} ux2 \\ uy2 \end{bmatrix} = \begin{bmatrix} 0.6 & 0.8 \\ -0.8 & 0.6 \end{bmatrix} \cdot \begin{bmatrix} uxp2 \\ uyp2 \end{bmatrix}$

#60: $\begin{bmatrix} ux2 \\ uy2 \end{bmatrix} = \begin{bmatrix} 0.6 \cdot uxp2+0.8 \cdot uyp2 \\ 0.6 \cdot uyp2-0.8 \cdot uxp2 \end{bmatrix}$

ux'2=0:

#61: $\begin{bmatrix} ux2 \\ uy2 \end{bmatrix} = \begin{bmatrix} 0.6 \cdot 0+0.8 \cdot uyp2 \\ 0.6 \cdot uyp2-0.8 \cdot 0 \end{bmatrix}$

#62: $uyp2=\frac{5\cdot ux2}{4} \wedge uyp2=\frac{5\cdot uy2}{3}$

#63: $\frac{5\cdot ux2}{4}=\frac{5\cdot uy2}{3}$

#64: $3\cdot ux2-4\cdot uy2=0$

En forma matricial:

#65: $$\begin{bmatrix} 3\cdot ux1+4\cdot uy1 \\ ux1-ux2 \\ 3\cdot ux2-4\cdot uy2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Usando el mismo vector de deformación, cuyo resultado es como la ecuación LEG (9-2):

#66: $$\begin{bmatrix} 3 & 4 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 3 & -4 & 0 \end{bmatrix}\cdot\begin{bmatrix} ux1 \\ uy1 \\ \theta z1 \\ ux2 \\ uy2 \\ \theta z2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Se reorganiza teniendo en cuenta que hay 3 variables independientes y 3 dependientes, que pueden separarse como [ux1,θx1,θx2]' independientes y [uy1,ux2,uy2]' independientes:

#67: $$\begin{bmatrix} 3 & 0 & 0 & 4 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 3 & -4 \end{bmatrix}\cdot\begin{bmatrix} ux1 \\ \theta z1 \\ \theta z2 \\ uy1 \\ ux2 \\ uy2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

#68: $\left[\begin{bmatrix}3&0&0\\1&0&0\\0&0&0\end{bmatrix}\cdot\begin{bmatrix}4&0&0\\0&-1&0\\0&3&-4\end{bmatrix}\right]\cdot\begin{bmatrix}\begin{bmatrix}ux1\\\theta z1\\\theta z2\end{bmatrix}\\\begin{bmatrix}uy1\\ux2\\uy2\end{bmatrix}\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$

Ecuación LEG (9-3):

#69: $[Ai,Ad]\cdot\begin{bmatrix}Ui\\Ud\end{bmatrix}=[[O]]$

#70: $\left[Ai:=\begin{bmatrix}3&0&0\\1&0&0\\0&0&0\end{bmatrix},Ad:=\begin{bmatrix}4&0&0\\0&-1&0\\0&3&-4\end{bmatrix}\right]$

#71: U:=[[ux1,θx1,θx2,uy1,ux2,uy2]]`

#72: $\left[Ui:=\begin{bmatrix}ux1\\\theta z1\\\theta z2\end{bmatrix},Ud:=\begin{bmatrix}uy1\\ux2\\uy2\end{bmatrix},O:=\begin{bmatrix}0\\0\\0\end{bmatrix}\right]$

Ro de la ecuación LEG (9-5):

#73: $Ro:=-Ad^{-1}\cdot Ai$

#74: $Ro:=-\begin{bmatrix}\frac{1}{4}&0&0\\0&-1&0\\0&-\frac{3}{4}&-\frac{1}{4}\end{bmatrix}\cdot Ai$

#75:

$$Ro:=\begin{bmatrix} -\frac{3}{4} & 0 & 0 \\ 1 & 0 & 0 \\ \frac{3}{4} & 0 & 0 \end{bmatrix}$$

R inicial de la ecuación LEG (9-6), con filas ordenadas conforme [U]=[Ui;Ud]=
[ux1,θz1,θz2,uy1,ux2,uy2]`:

#76:

$$Rini:=\begin{bmatrix} \text{IDENTITY\_MATRIX}(3) \\ [Ro] \end{bmatrix}$$

#77:

$$Rini:=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\frac{3}{4} & 0 & 0 \\ 1 & 0 & 0 \\ \frac{3}{4} & 0 & 0 \end{bmatrix}$$

Reordenar R para que tenga el orden de U:

#78:

$$R:=\left[Rini_1, Rini_4, Rini_2, Rini_5, Rini_6, Rini_3\right]$$

#79:

$$R:=\begin{bmatrix} 1 & 0 & 0 \\ -\frac{3}{4} & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ \frac{3}{4} & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Ecuación LEG (9-7):

#80: $U=R\cdot Ui$

#81:

$$\begin{bmatrix} ux1 \\ \theta x1 \\ \theta x2 \\ uy1 \\ ux2 \\ uy2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\frac{3}{4} & 0 & 0 \\ 1 & 0 & 0 \\ \frac{3}{4} & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} ux1 \\ \theta z1 \\ \theta z2 \end{bmatrix}$$

#82: $Ui=R'\cdot U$

#83:

$$\begin{bmatrix} ux1 \\ \theta z1 \\ \theta z2 \end{bmatrix} = \begin{bmatrix} ux1+ux2-\frac{3\cdot uy1}{4}+\frac{3\cdot uy2}{4} \\ \theta z1 \\ \theta z2 \end{bmatrix}$$

#84:

$$R'=\begin{bmatrix} 1 & 0 & 0 & -\frac{3}{4} & 1 & \frac{3}{4} \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

#85:

$$\begin{bmatrix} ux1 \\ \theta z1 \\ \theta z2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -\frac{3}{4} & 1 & \frac{3}{4} \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} ux1 \\ \theta z1 \\ \theta z2 \\ uy1 \\ ux2 \\ uy2 \end{bmatrix}$$

Ecuación matricial reducida {fuerzas}=[K]*{deformación}:

#86: $F=Kr\cdot U$

#87: $$F=\begin{bmatrix} \frac{2\cdot(17\cdot\psi+96)}{25} & \frac{12\cdot(\psi-12)}{25} & \frac{24}{5} & -\psi & 0 & 0 \\ \frac{12\cdot(\psi-12)}{25} & \frac{8\cdot(2\cdot\psi+51)}{25} & \frac{12}{5} & 0 & -12 & 6 \\ \frac{24}{5} & \frac{12}{5} & 8 & 0 & -6 & 2 \\ -\psi & 0 & 0 & \frac{2\cdot(17\cdot\psi+96)}{25} & \frac{12\cdot(12-\psi)}{25} & \frac{24}{5} \\ 0 & -12 & -6 & \frac{12\cdot(12-\psi)}{25} & \frac{8\cdot(2\cdot\psi+51)}{25} & -\frac{12}{5} \\ 0 & 6 & 2 & \frac{24}{5} & -\frac{12}{5} & 8 \end{bmatrix}\cdot\begin{bmatrix} ux1 \\ \theta z1 \\ \theta z2 \\ uy1 \\ ux2 \\ uy2 \end{bmatrix}$$

Contragradiente a las fuerzas de las deformaciones independientes:

#88: $Fi=R`\cdot F$

Usando la ecuación matricial:

#89: $Fi=R`\cdot(Kr\cdot U)$

Usando $U=R\cdot Ui$:

#90: $Fi=R`\cdot Kr\cdot(R\cdot Ui)$

Organizando;

#91: $Fi=(R`\cdot Kr\cdot R)\cdot Ui$

#92: $Fi=Ki\cdot Ui$

#93: $Ki:=R`\cdot Kr\cdot R$

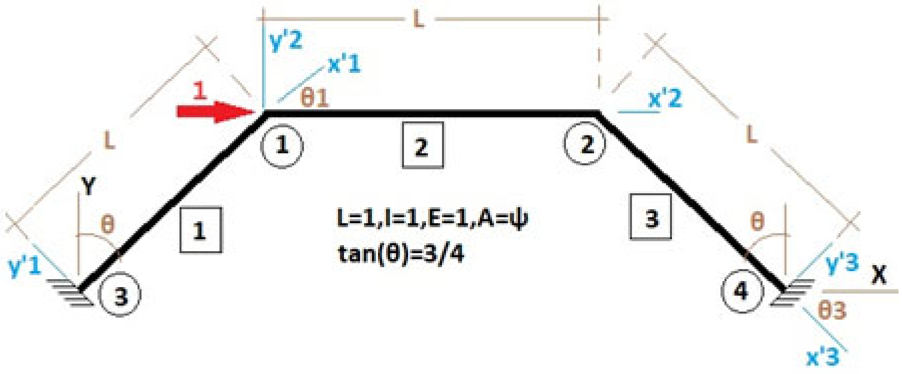
#94: $$Ki:=\frac{1}{25}\cdot\begin{bmatrix} 1612.5 & -37.5 & -37.5 \\ -37.5 & 200 & 50 \\ -37.5 & 50 & 200 \end{bmatrix}$$

La matriz de rigidez para operar los gdl independientes no depende del área, expresada como ψ, tal como se procedió a establecer que los elementos tienen una rigidez axial infinita.

#95: Fi=Ki·Ui

#96: $$Fi=\frac{1}{25}\cdot\begin{bmatrix}1612.5 & -37.5 & -37.5\\ -37.5 & 200 & 50\\ -37.5 & 50 & 200\end{bmatrix}\cdot\begin{bmatrix}ux1\\ \theta z1\\ \theta z2\end{bmatrix}$$

Aplicar F1x=1:



#97: $$\begin{bmatrix}Fx1\\ M1\\ M2\end{bmatrix}=\frac{1}{25}\cdot\begin{bmatrix}1612.5 & -37.5 & -37.5\\ -37.5 & 200 & 50\\ -37.5 & 50 & 200\end{bmatrix}\cdot\begin{bmatrix}ux1\\ \theta z1\\ \theta z2\end{bmatrix}$$

#98: $$\begin{bmatrix}1\\ 0\\ 0\end{bmatrix}=\frac{1}{25}\cdot\begin{bmatrix}1612.5 & -37.5 & -37.5\\ -37.5 & 200 & 50\\ -37.5 & 50 & 200\end{bmatrix}\cdot\begin{bmatrix}ux1\\ \theta z1\\ \theta z2\end{bmatrix}$$

#99: $$\left(\frac{1}{25}\cdot\begin{bmatrix}1612.5 & -37.5 & -37.5\\ -37.5 & 200 & 50\\ -37.5 & 50 & 200\end{bmatrix}\right)^{-1}\cdot\begin{bmatrix}1\\ 0\\ 0\end{bmatrix}=\begin{bmatrix}ux1\\ \theta z1\\ \theta z2\end{bmatrix}$$

#100: $$\begin{bmatrix}\frac{20}{1281}\\ \frac{1}{427}\\ \frac{1}{427}\end{bmatrix}=\begin{bmatrix}ux1\\ \theta z1\\ \theta z2\end{bmatrix}$$

#101: $$ux1=\frac{20}{1281}\wedge \theta x1=\frac{1}{427}\wedge \theta x2=\frac{1}{427}$$