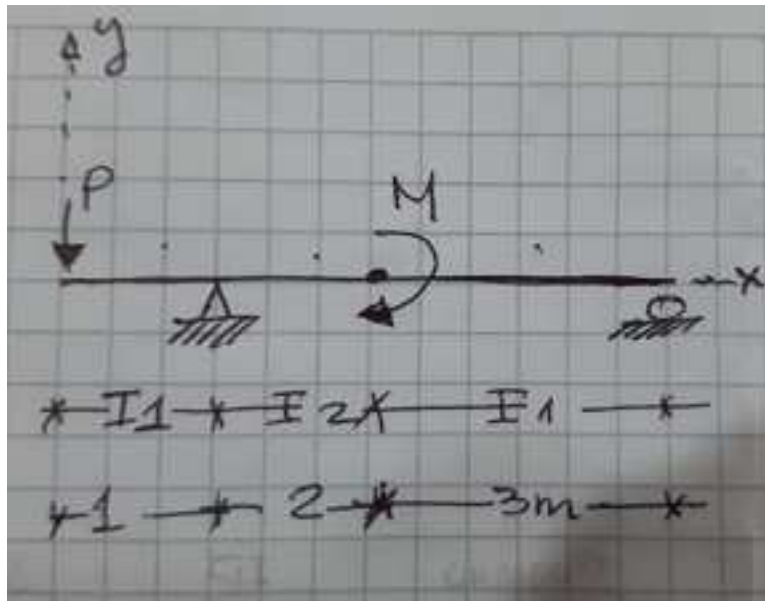


Determinar las funciones que representan la **curva elástica** por flexión pura de la viga de la figura (utilizar el método de la **Carga Unitaria Ficticia**):



#1: [CaseMode := Sensitive, InputMode := Word]

#2: $[P := 5, M := 2, E := 2 \cdot 10^7, I1 := 1.9 \cdot 10^{-4}, I2 := 2.2 \cdot 10^{-4}]$

Equilibrio estático:

#3: [R1 :=, R2 :=]

#4:
$$\begin{bmatrix} R1 + R2 - P = 0 \\ R2 \cdot 5 - M + P \cdot 1 = 0 \end{bmatrix}$$

#5:
$$\left[R1 := \frac{28}{5}, R2 := -\frac{3}{5} \right]$$

#6: $[R1 = 5.6 \wedge R2 = -0.6]$

Momentos flectores: MF1($0 \leq x \leq 1$), MF2($1 \leq x \leq 3$), MF3($3 \leq x \leq 6$)

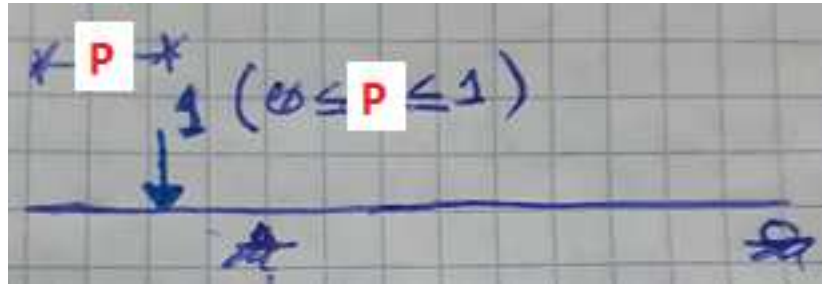
#7: [MF1(x) :=, MF2(x) :=, MF3(x) :=]

#8:
$$\begin{bmatrix} MF1(x) := - P \cdot x \\ MF2(x) := - P \cdot x + R1 \cdot (x - 1) \\ MF3(x) := - P \cdot x + R1 \cdot (x - 1) + M \end{bmatrix}$$

#9:

$$\begin{bmatrix} MF1(x) := -5 \cdot x \\ MF2(x) := 0.6 \cdot x - 5.6 \\ MF3(x) := 0.6 \cdot x - 3.6 \end{bmatrix}$$

I) Aplicación de la carga unitaria ficticia entre $0 \leq p \leq 1$:



Equilibrio estático:

#10: $[r1I :=, r2I :=]$

#11:
$$\begin{bmatrix} r1I + r2I - 1 = 0 \\ r2I \cdot 5 + 1 \cdot (1 - p) = 0 \end{bmatrix}$$

#12: $[r1I := 1.2 - 0.2 \cdot p, r2I := 0.2 \cdot p - 0.2]$

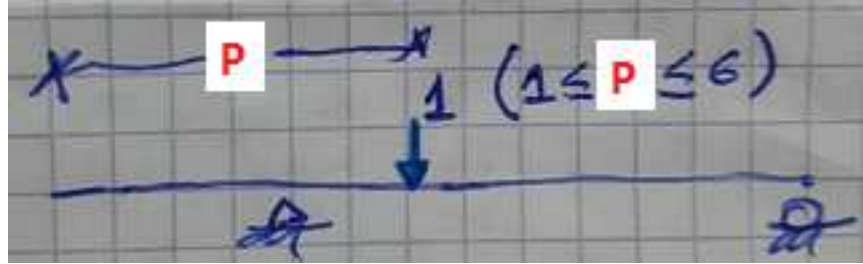
Momentos flectores de la viga con carga unitaria ficticia: $mf1(0 \leq x \leq p), mf2(p \leq x \leq 1), mf3(1 \leq x \leq 6)$

#13: $[mf1I(x) :=, mf2I(x) :=, mf3I(x) :=]$

#14:
$$\begin{bmatrix} mf1I(x) := 0 \\ mf2I(x) := -1 \cdot (x - p) \\ mf3I(x) := -1 \cdot (x - p) + r1I \cdot (x - 1) \end{bmatrix}$$

#15:
$$\begin{bmatrix} mf1I(x) := 0 \\ mf2I(x) := p - x \\ mf3I(x) := x \cdot (0.2 - 0.2 \cdot p) + 1.2 \cdot p - 1.2 \end{bmatrix}$$

II) Aplicación de la carga unitaria ficticia entre $1 \leq p \leq 6$:



Equilibrio estático:

#16: [r1II :=, r2II :=]

#17:
$$\begin{bmatrix} r1II + r2II - 1 = 0 \\ r2II \cdot 5 - 1 \cdot (p - 1) = 0 \end{bmatrix}$$

#18: [r1II := 1.2 - 0.2 · p, r2II := 0.2 · p - 0.2]

Momentos flectores de la viga con carga unitaria ficticia: mf1(0 ≤ x ≤ 1), mf2(1 ≤ x ≤ p), mf3 (p ≤ x ≤ 6)

#19: [mf1II(x) :=, mf2II(x) :=, mf3II(x) :=]

#20:
$$\begin{bmatrix} mf1II(x) := 0 \\ mf2II(x) := r1II \cdot (x - 1) \\ mf3II(x) := r1II \cdot (x - 1) - 1 \cdot (x - p) \end{bmatrix}$$

#21:
$$\begin{bmatrix} mf1II(x) := 0 \\ mf2II(x) := 0.2 \cdot x \cdot (6 - p) + 0.2 \cdot (p - 6) \\ mf3II(x) := 0.2 \cdot x \cdot (1 - p) + 1.2 \cdot (p - 1) \end{bmatrix}$$

Cálculo de las funciones de deflexión: Δ1(0 ≤ p ≤ 1), Δ(1 ≤ p ≤ 3), Δ(3 ≤ p ≤ 6)

#22: [Δ1(p) :=, Δ2(p) :=, Δ3(p) :=]

#23:
$$\begin{bmatrix} \Delta1(p) := \frac{1}{E} \cdot \left(\frac{1}{I1} \cdot \int_0^p MF1(x) \cdot mf1II(x) dx + \frac{1}{I1} \cdot \int_p^1 MF1(x) \cdot mf2II(x) dx + \right. \\ \Delta2(p) := \frac{1}{E} \cdot \left(\frac{1}{I1} \cdot \int_0^1 MF1(x) \cdot mf1II(x) dx + \frac{1}{I2} \cdot \int_1^p MF2(x) \cdot mf2II(x) dx + \right. \\ \left. \Delta3(p) := \frac{1}{E} \cdot \left(\frac{1}{I1} \cdot \int_0^1 MF1(x) \cdot mf1II(x) dx + \frac{1}{I2} \cdot \int_1^3 MF2(x) \cdot mf2II(x) dx + \right. \end{bmatrix}$$

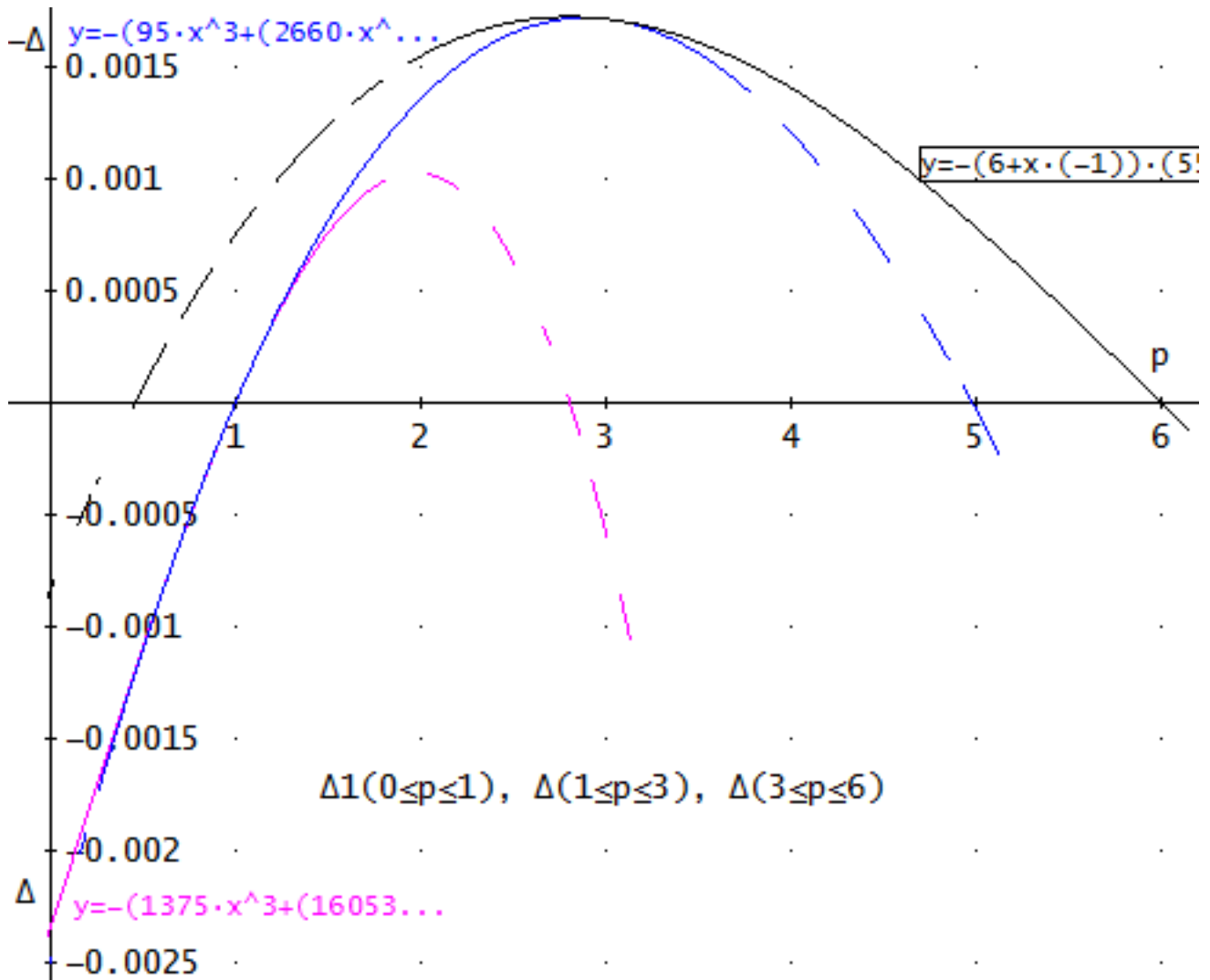
$$\left. \begin{aligned} & \frac{1}{I2} \cdot \int_1^3 MF2(x) \cdot mf3I(x) \, dx + \frac{1}{I1} \cdot \int_3^6 MF3(x) \cdot mf3I(x) \, dx \right\} \\ & \frac{1}{I2} \cdot \int_p^3 MF2(x) \cdot mf3II(x) \, dx + \frac{1}{I1} \cdot \int_3^6 MF3(x) \cdot mf3II(x) \, dx \right\} \\ & \frac{1}{I1} \cdot \int_3^p MF3(x) \cdot mf2II(x) \, dx + \frac{1}{I1} \cdot \int_p^6 MF3(x) \cdot mf3II(x) \, dx \right\} \end{aligned}$$

#24:

$$\left[\begin{aligned} \Delta1(p) &:: \frac{1375 \cdot p^3 - 16053 \cdot p + 14678}{6270000} \\ \Delta2(p) &:: - \frac{95 \cdot p^3 - 2660 \cdot p^2 + 12987 \cdot p - 10422}{4180000} \\ \Delta3(p) &:: \frac{(6 - p) \cdot (55 \cdot p^2 - 660 \cdot p + 291)}{2090000} \end{aligned} \right]$$

#25:

$$\left[\begin{aligned} \Delta1(p) &:: 0.0002192982456 \cdot (p^3 - 3 \cdot p + 2) \\ \Delta2(p) &:: - 4.545454545 \cdot 10^{-6} \cdot (5 \cdot p^3 - 140 \cdot p^2 + 621 \cdot p - 486) \\ \Delta3(p) &:: 5.263157894 \cdot 10^{-6} \cdot (6 - p) \cdot (5 \cdot p^2 - 60 \cdot p + 99) \end{aligned} \right]$$



La deflexión máxima positiva (hacia abajo) se presenta en el voladizo, en $\Delta_1(0)$ y la deflexión máxima negativa (hacia arriba) se presenta en un punto cercano a $p=3m$, sin especificar a priori la Δ_2 y Δ_3 :

#26: $\Delta_1(0) = 0.002340988835$

#27: $\frac{d}{dp} \Delta_2(p) = 0$

Puntos de deflexión máxima para la función $\Delta_2(1 \leq p \leq 3)$:

#28: $\frac{d}{dp} \Delta_2(p) = 0$

#29: $p = 2.887971429$

#30: $\Delta_2(2.887971429) = -0.001719358998$

Puntos de deflexión máxima para la función $\Delta_3(3 \leq p \leq 6)$:

$$\#31: \frac{d}{dp} \Delta_3(p) = 0$$

$$\#32: p \cdot (p - 12) = -25.76363636$$

$$\#33: p = 2.800568231 \vee p = 9.199431768$$

$$\#34: \Delta_3(3) = -0.001713875598$$

El punto de deflexión máxima negativa (hacia arriba) está en 2.88m con 1.719 mm