

#1: [CaseMode := Sensitive, InputMode := Word]

Datos del ejercicio:

#2:
$$\left[L1 := 4, L2 := 3, L3 := 2, E := 2 \cdot 10^8, I_v := \frac{0.2^4}{12}, I_c := \frac{0.15^4}{12}, P := 10, W := 5 \right]$$

Simplificación de expresiones de EI:

#3: [EIc := E·Ic, EIV := E·IV]

Incognitas a encontrar:

#4: [x1 :=, y1 :=, y3 :=]

Ecuaciones de la estática:

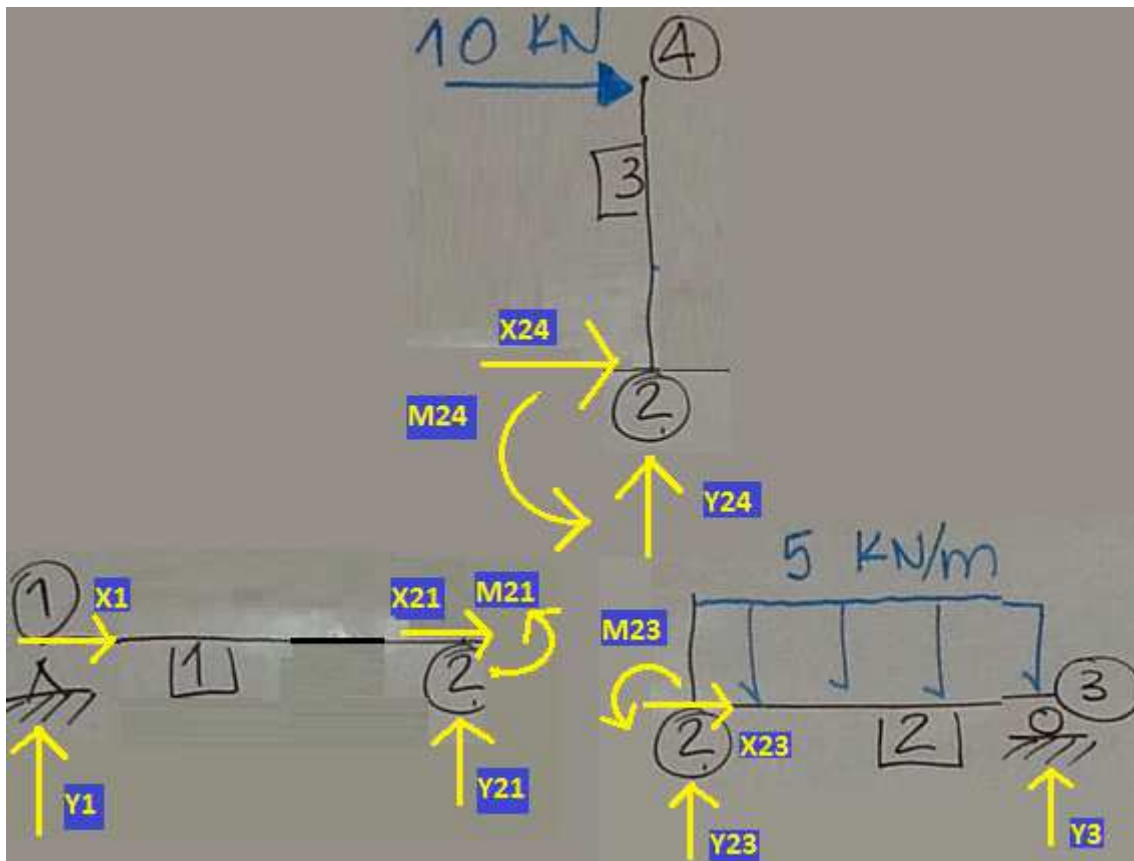
#5:
$$\left[\begin{array}{l} x1 + P = 0 \\ y1 + y3 - W \cdot L2 = 0 \\ y3 \cdot (L1 + L2) - P \cdot L3 - W \cdot L2 \cdot \left(L1 + \frac{L2}{2} \right) \end{array} \right]$$

#6:
$$\left[x1 := -10, y1 := \frac{5}{14}, y3 := \frac{205}{14} \right]$$

#7: [x1 := -10, y1 := 0.3571428571, y3 := 14.64285714]

Diagramas de cuerpo libre:

Elemento 1:



Valores a encontrar (fuerzas y momentos internos):

#8:
$$\begin{bmatrix} M21 :: M23 :: M24 :: \\ x21 :: x23 :: x24 :: \\ y21 :: y23 :: y24 :: \end{bmatrix}$$

Ecuaciones de la estática:

#9:
$$\begin{bmatrix} x1 + x21 = 0 & y1 + y21 = 0 & y21 \cdot L1 + M21 = 0 \\ x23 = 0 & y23 + y3 - W \cdot L2 = 0 & M23 + y3 \cdot L2 - W \cdot L2 \cdot \frac{L2}{2} = 0 \\ x24 + P = 0 & y24 = 0 & M24 - P \cdot L3 = 0 \end{bmatrix}$$

#10:
$$\begin{bmatrix} M21 :: \frac{10}{7} & M23 :: -\frac{150}{7} & M24 :: 20 \\ x21 :: 10 & x23 :: 0 & x24 :: -10 \\ y21 :: -\frac{5}{14} & y23 :: \frac{5}{14} & y24 :: 0 \end{bmatrix}$$

Chequeo estático del nudo 2:

#11:
$$\begin{bmatrix} M21 + M23 + M24 = 0 \\ x21 + x23 + x24 = 0 \\ y21 + y23 + y24 = 0 \end{bmatrix} = \begin{bmatrix} \text{true} \\ \text{true} \\ \text{true} \end{bmatrix}$$

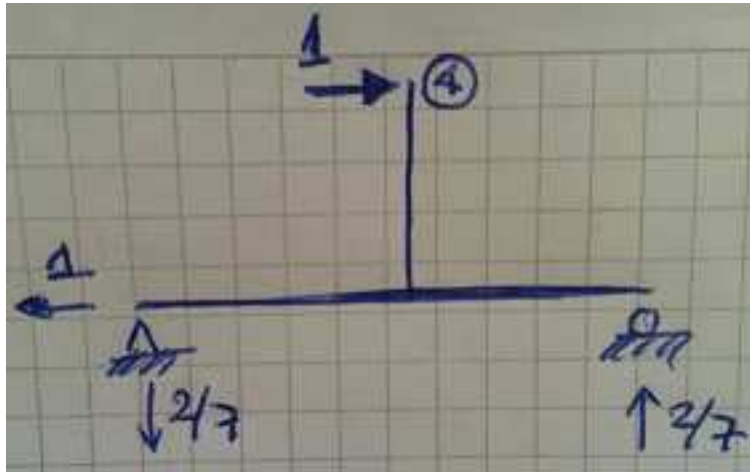
Momentos flectores del estado de cargas original:

#12: [MF1(x) :=, MF2(x) :=, MF3(x) :=]

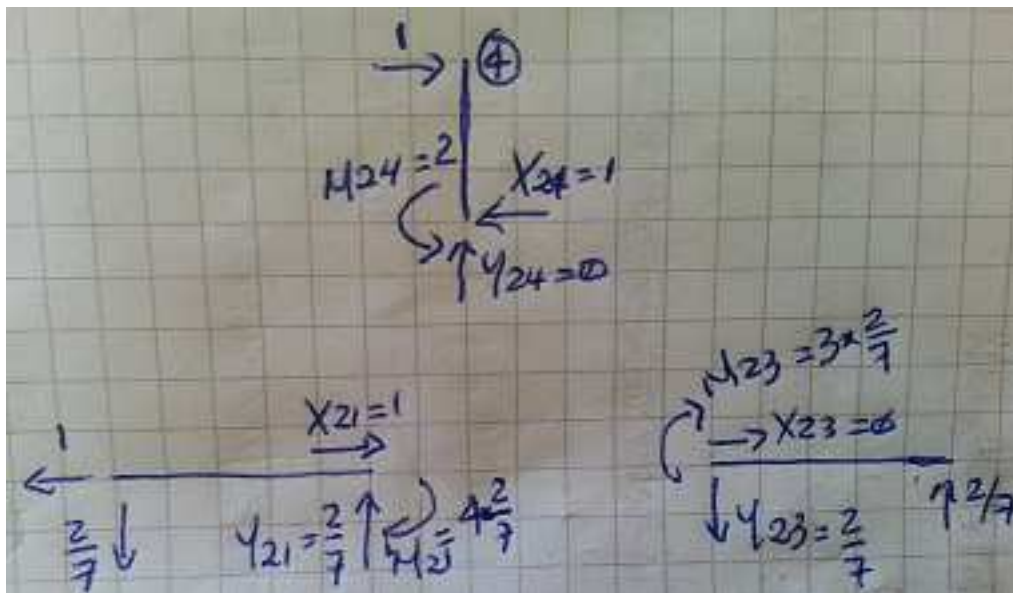
$$\begin{bmatrix} \text{MF1}(x) := y_1 \cdot x \\ \text{MF2}(x) := -M_{23} + y_{23} \cdot x - W \cdot x \cdot \frac{x}{2} \\ \text{MF3}(x) := P \cdot x \end{bmatrix}$$

$$\begin{bmatrix} \text{MF1}(x) := \frac{5 \cdot x}{14} \\ \text{MF2}(x) := -\frac{5 \cdot x^2}{2} + \frac{5 \cdot x}{14} + \frac{150}{7} \\ \text{MF3}(x) := 10 \cdot x \end{bmatrix}$$

Deformación del nudo 4 en el eje x (horizontal):



Diagramas de cuerpo libre:



Momentos flectores de la estructura cargada con la carga unitaria ficticia:

$$\#15: \begin{bmatrix} mf1(x) := -\frac{2}{7} \cdot x \\ mf2(x) := 3 \cdot \frac{2}{7} - \frac{2}{7} \cdot x \\ mf3(x) := 1 \cdot x \end{bmatrix}$$

$$\#16: \begin{bmatrix} mf1(x) := -\frac{2 \cdot x}{7} \\ mf2(x) := \frac{6}{7} - \frac{2 \cdot x}{7} \\ mf3(x) := x \end{bmatrix}$$

Deformación horizontal del nudo 4:

$$\#17: d4x = \frac{1}{EIv} \cdot \int_0^{L1} MF1(x) \cdot mf1(x) dx + \frac{1}{EIv} \cdot \int_0^{L2} MF2(x) \cdot mf2(x) dx + \frac{1}{EIc} \cdot \int_0^{L3} MF3(x) \cdot mf3(x) dx$$

$$\#18: d4x = \frac{3}{80000} \cdot \int_0^4 \frac{5 \cdot x}{14} \cdot \left(-\frac{2 \cdot x}{7}\right) dx + \frac{3}{80000} \cdot \int_0^3 \left(-\frac{5 \cdot x^2}{2} + \frac{5 \cdot x}{14} + \frac{150}{7}\right) \cdot \left(\frac{6}{7} - \frac{2 \cdot x}{7}\right) dx +$$

$$\frac{2}{16875} \cdot \int_0^2 (10 \cdot x) \cdot x dx$$

$$\#19: d4x = \frac{3}{80000} \cdot \int_0^4 (0.3571428571 \cdot x) \cdot (-0.2857142857 \cdot x) dx + \frac{3}{80000} \cdot \int_0^3 (-2.5 \cdot x^2 + 0.3571428571 \cdot x +$$

$$21.42857142) \cdot (0.8571428571 - 0.2857142857 \cdot x) dx + \frac{2}{16875} \cdot \int_0^2 (10 \cdot x) \cdot x dx$$

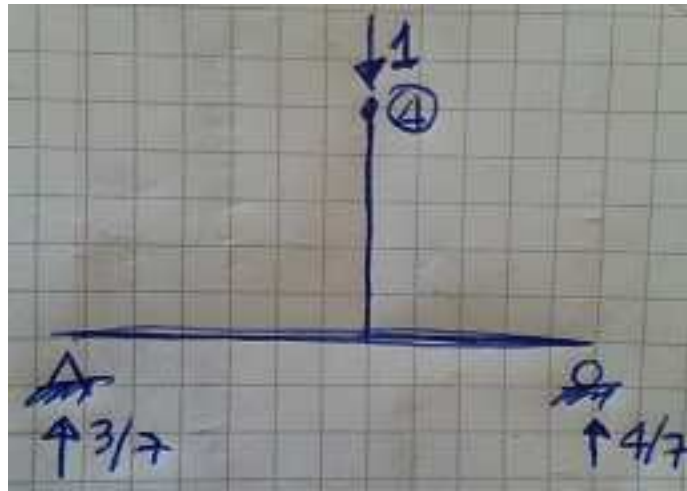
$$\#20: d4x = 0.003948440255$$

Si la columna tiene un eje x ascendente hacia arriba con el mismo resultado final:

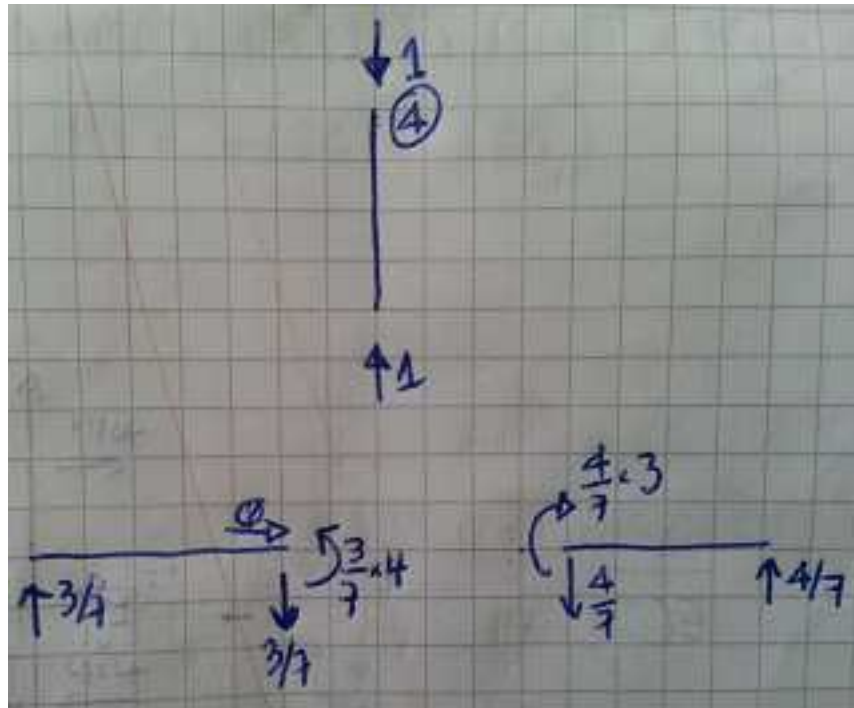
$$\#21: [MF3(x) := -M24 - x24 \cdot x, mf3(x) := -2 + x]$$

$$\#22: d4x = \frac{1}{EIv} \cdot \int_0^{L1} MF1(x) \cdot mf1(x) dx + \frac{1}{EIv} \cdot \int_0^{L2} MF2(x) \cdot mf2(x) dx + \frac{1}{EIc} \cdot \int_0^2 (10 \cdot x - 20) \cdot (x - 2) dx$$

Deformación del nudo 4 en el eje y (vertical):



Diagramas de cuerpo libre:



Momentos flectores de la estructura cargada con la carga unitaria ficticia:

$$\#23: \begin{bmatrix} mf1(x) := \frac{3}{7} \cdot x \\ mf2(x) := 3 \cdot \frac{4}{7} - \frac{4}{7} \cdot x \\ mf3(x) := 0 \end{bmatrix}$$

#24:

$$\begin{bmatrix} mf1(x) := \frac{3 \cdot x}{7} \\ mf2(x) := \frac{12}{7} - \frac{4 \cdot x}{7} \\ mf3(x) := 0 \end{bmatrix}$$

Deformación vertical del nudo 4:

#25:
$$d4y = \frac{1}{EI_v} \cdot \int_0^{L1} MF1(x) \cdot mf1(x) dx + \frac{1}{EI_v} \cdot \int_0^{L2} MF2(x) \cdot mf2(x) dx + \frac{1}{EI_c} \cdot \int_0^{L3} MF3(x) \cdot mf3(x) dx$$

#26:
$$d4y = \frac{3}{80000} \cdot \int_0^4 \frac{5 \cdot x}{14} \cdot \frac{3 \cdot x}{7} dx + \frac{3}{80000} \cdot \int_0^3 \left(-\frac{5 \cdot x^2}{2} + \frac{5 \cdot x}{14} + \frac{150}{7} \right) \cdot \left(\frac{12}{7} - \frac{4 \cdot x}{7} \right) dx +$$

$$\frac{2}{16875} \cdot \int_0^2 (10 \cdot x) \cdot 0 dx$$

#27:

$d4y = 0.001861607142$

Deformación haciendo uso de un programa comercial (Sap2000):

