

[CaseMode := Sensitive, InputMode := Word]

$$\left[ \begin{array}{ccc} & & 4 \\ & 7 & 0.4 \\ E := 2.1 \cdot 10^7 & , \text{Icol} := \frac{0.4}{12} & , \text{Ivig} := \frac{0.3}{12} \end{array} \right]$$

Determinar los desplazamientos hacia la derecha de los nudos 3, 4 y 5, utilizando el método de la **Carga Unitaria Ficticia**:

Equilibrio estático:

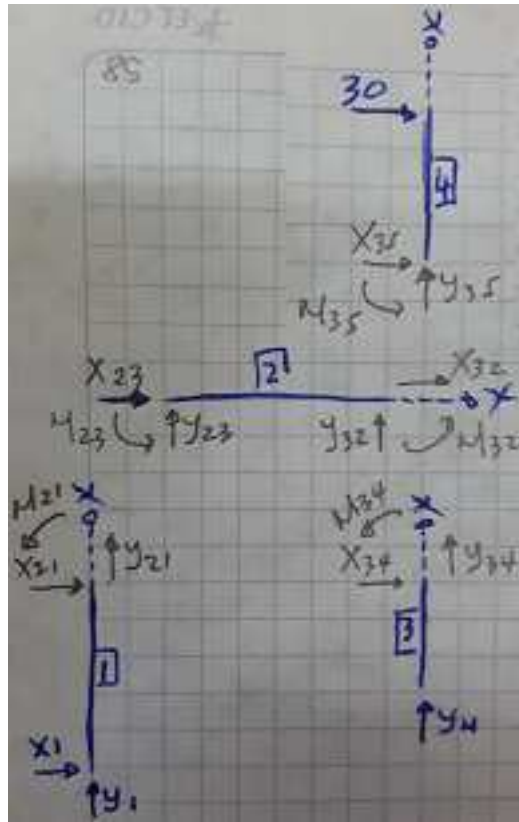
[X1 :=, Y1 :=, Y4 :=]

$$\left[ \begin{array}{l} X1 + 40 + 30 = 0 \\ Y1 + Y4 = 0 \\ Y4 \cdot 5 - 40 \cdot 4 - 30 \cdot 7 = 0 \end{array} \right]$$

$$[X1 := -70, Y1 := -74, Y4 := 74]$$

Diagramas de cuerpo libre (DCL):

$$\left[ \begin{array}{l} X21 := Y21 := M21 := \\ X23 := Y23 := M23 := \\ X32 := Y32 := M32 := \\ X34 := Y34 := M34 := \\ X35 := Y35 := M35 := \end{array} \right]$$



Secuencia: elemento e1, nudo n2, e3, e4, n3:

$$\begin{bmatrix} X_{21} + X_1 = 0 & Y_{21} + Y_1 = 0 & M_{21} + X_1 \cdot 4 = 0 \\ X_{21} + X_{23} = 40 & Y_{21} + Y_{23} = 0 & M_{21} + M_{23} = 0 \\ X_{34} = 0 & Y_4 + Y_{34} = 0 & M_{34} = 0 \\ X_{35} + 30 = 0 & Y_{35} = 0 & M_{35} - 30 \cdot 3 = 0 \\ X_{32} + X_{34} + X_{35} = 0 & Y_{32} + Y_{34} + Y_{35} = 0 & M_{32} + M_{34} + M_{35} = 0 \end{bmatrix}$$

$$\begin{bmatrix} M_{21} := 280 & M_{23} := -280 & M_{32} := -90 & M_{34} := 0 & M_{35} := 90 \\ X_{21} := 70 & X_{23} := -30 & X_{32} := 30 & X_{34} := 0 & X_{35} := -30 \\ Y_{21} := 74 & Y_{23} := -74 & Y_{32} := 74 & Y_{34} := -74 & Y_{35} := 0 \end{bmatrix}$$

Chequeo en elemento 2:

$$[X_{23} + X_{32} = 0, Y_{23} + Y_{32} = 0, M_{23} + M_{32} + Y_{32} \cdot 5 = 0]$$

[true, true, true]

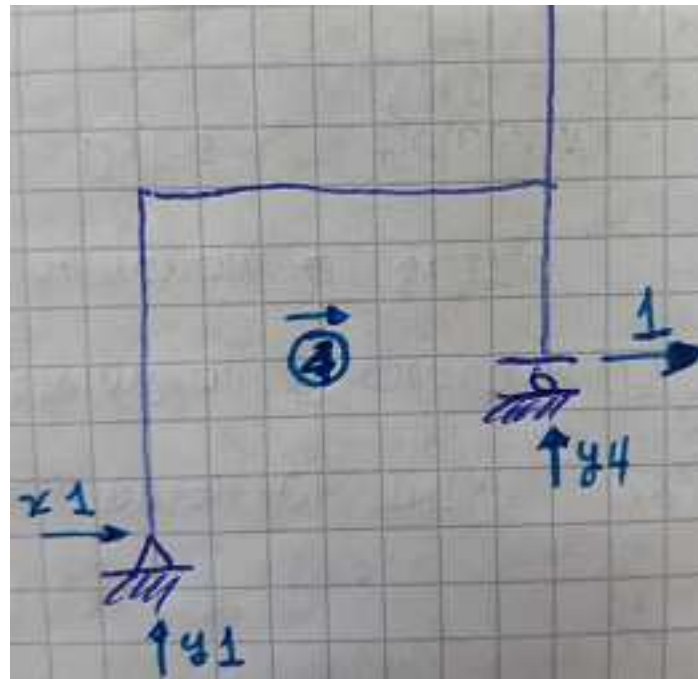
Funciones de **momento flector** en cada elemento individual:

$$[MF1(x) :=, MF2(x) :=, MF3(x) :=, MF4(x) :=]$$

$$\begin{bmatrix} MF1(x) := -X1 \cdot x \\ MF2(x) := -M23 + Y23 \cdot x \\ MF3(x) := 0 \\ MF4(x) := -M35 - X35 \cdot x \end{bmatrix}$$

$$\begin{bmatrix} MF1(x) := 70 \cdot x \\ MF2(x) := 280 - 74 \cdot x \\ MF3(x) := 0 \\ MF4(x) := 30 \cdot x - 90 \end{bmatrix}$$

Aplicación de una **carga unitaria ficticia en el nudo 4:**



Equilibrio estático:

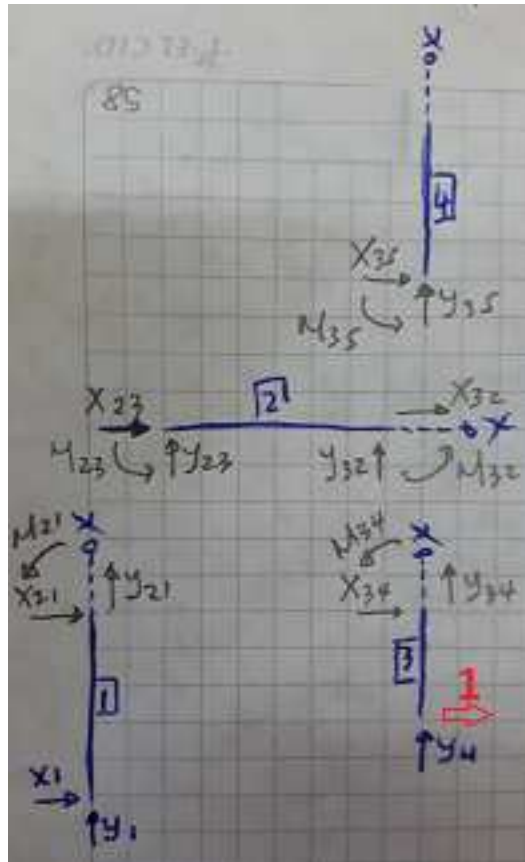
$$[x1n4 :=, y1n4 :=, y4n4 :=]$$

$$\begin{bmatrix} x1n4 + 1 = 0 \\ y1n4 + y4n4 = 0 \\ y4n4 \cdot 5 - 1 \cdot 1 = 0 \end{bmatrix}$$

$$[x1n4 := -1, y1n4 := -0.2, y4n4 := 0.2]$$

Diagramas de cuerpo libre:

$$\begin{bmatrix} x_{21n4} := & y_{21n4} := & m_{21n4} := \\ x_{23n4} := & y_{23n4} := & m_{23n4} := \\ x_{32n4} := & y_{32n4} := & m_{32n4} := \\ x_{34n4} := & y_{34n4} := & m_{34n4} := \\ x_{35n4} := & y_{35n4} := & m_{35n4} := \end{bmatrix}$$



Secuencia: elemento e1, nudo n2, e3, e4, n3:

$$\begin{bmatrix} x_{21n4} + x_{1n4} = 0 & y_{21n4} + y_{1n4} = 0 & m_{21n4} + x_{1n4} \cdot 4 = 0 \\ x_{21n4} + x_{23n4} = 0 & y_{21n4} + y_{23n4} = 0 & m_{21n4} + m_{23n4} = 0 \\ x_{34n4} + 1 = 0 & y_{4n4} + y_{34n4} = 0 & m_{34n4} + 1 \cdot 3 = 0 \\ x_{35n4} = 0 & y_{35n4} = 0 & m_{35n4} = 0 \\ x_{32n4} + x_{34n4} + x_{35n4} = 0 & y_{32n4} + y_{34n4} + y_{35n4} = 0 & m_{32n4} + m_{34n4} + m_{35n4} = 0 \end{bmatrix}$$

$$\begin{bmatrix} m_{21n4} := 4 & m_{23n4} := -4 & m_{32n4} := 3 & m_{34n4} := -3 & m_{35n4} := 0 \\ x_{21n4} := 1 & x_{23n4} := -1 & x_{32n4} := 1 & x_{34n4} := -1 & x_{35n4} := 0 \\ y_{21n4} := 0.2 & y_{23n4} := -0.2 & y_{32n4} := 0.2 & y_{34n4} := -0.2 & y_{35n4} := 0 \end{bmatrix}$$

Chequeo en elemento 2:

$$[x_{23n4} + x_{32n4} = 0, y_{23n4} + y_{32n4} = 0, m_{23n4} + m_{32n4} + y_{32n4} \cdot 5 = 0]$$

$$[true, true, true]$$

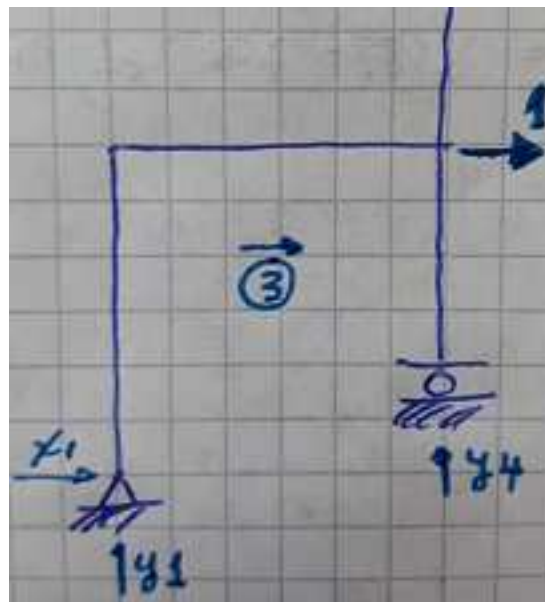
Funciones de momento flector:

$$[mf_{1n4}(x) :=, mf_{2n4}(x) :=, mf_{3n4}(x) :=, mf_{4n4}(x) :=]$$

$$\begin{bmatrix} mf_{1n4}(x) := -x_{1n4} \cdot x \\ mf_{2n4}(x) := -m_{23n4} + y_{23n4} \cdot x \\ mf_{3n4}(x) := -1 \cdot x \\ mf_{4n4}(x) := -m_{35n4} - x_{35n4} \cdot x \end{bmatrix}$$

$$\begin{bmatrix} mf_{1n4}(x) := x \\ mf_{2n4}(x) := 4 - 0.2 \cdot x \\ mf_{3n4}(x) := -x \\ mf_{4n4}(x) := 0 \end{bmatrix}$$

Aplicación de una **carga unitaria ficticia en el nudo 3**:



Equilibrio estático:

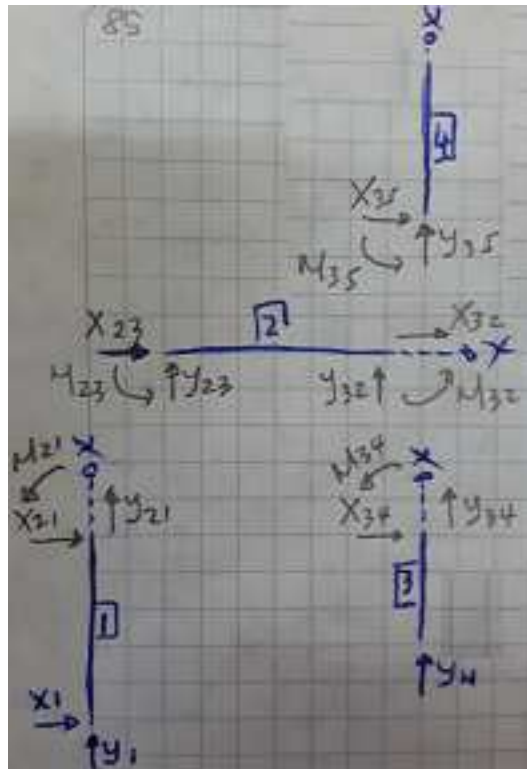
[x1n3 :=, y1n3 :=, y4n3 :=]

$$\begin{bmatrix} x1n3 + 1 = 0 \\ y1n3 + y4n3 = 0 \\ y4n3 \cdot 5 - 1 \cdot 4 = 0 \end{bmatrix}$$

[x1n3 := -1, y1n3 := -0.8, y4n3 := 0.8]

Diagramas de cuerpo libre:

$$\begin{bmatrix} x21n3 := & y21n3 := & m21n3 := \\ x23n3 := & y23n3 := & m23n3 := \\ x32n3 := & y32n3 := & m32n3 := \\ x34n3 := & y34n3 := & m34n3 := \\ x35n3 := & y35n3 := & m35n3 := \end{bmatrix}$$



Secuencia: elemento e1, nudo n2, e3, e4, n3:

$$\left[ \begin{array}{ccc} x_{21n3} + x_{1n3} = 0 & y_{21n3} + y_{1n3} = 0 & m_{21n3} + x_{1n3} \cdot 4 = 0 \\ x_{21n3} + x_{23n3} = 0 & y_{21n3} + y_{23n3} = 0 & m_{21n3} + m_{23n3} = 0 \\ x_{34n3} = 0 & y_{4n3} + y_{34n3} = 0 & m_{34n3} = 0 \\ x_{35n3} = 0 & y_{35n3} = 0 & m_{35n3} = 0 \\ x_{32n3} + x_{34n3} + x_{35n3} = 1 & y_{32n3} + y_{34n3} + y_{35n3} = 0 & m_{32n3} + m_{34n3} + m_{35n3} = 0 \end{array} \right]$$

$$\left[ \begin{array}{ccccc} m_{21n3} := 4 & m_{23n3} := -4 & m_{32n3} := 0 & m_{34n3} := 0 & m_{35n3} := 0 \\ x_{21n3} := 1 & x_{23n3} := -1 & x_{32n3} := 1 & x_{34n3} := 0 & x_{35n3} := 0 \\ y_{21n3} := 0.8 & y_{23n3} := -0.8 & y_{32n3} := 0.8 & y_{34n3} := -0.8 & y_{35n3} := 0 \end{array} \right]$$

Chequeo en elemento 2:

$$[x_{23n3} + x_{32n3} = 0, y_{23n3} + y_{32n3} = 0, m_{23n3} + m_{32n3} + y_{32n3} \cdot 5 = 0]$$

[true, true, true]

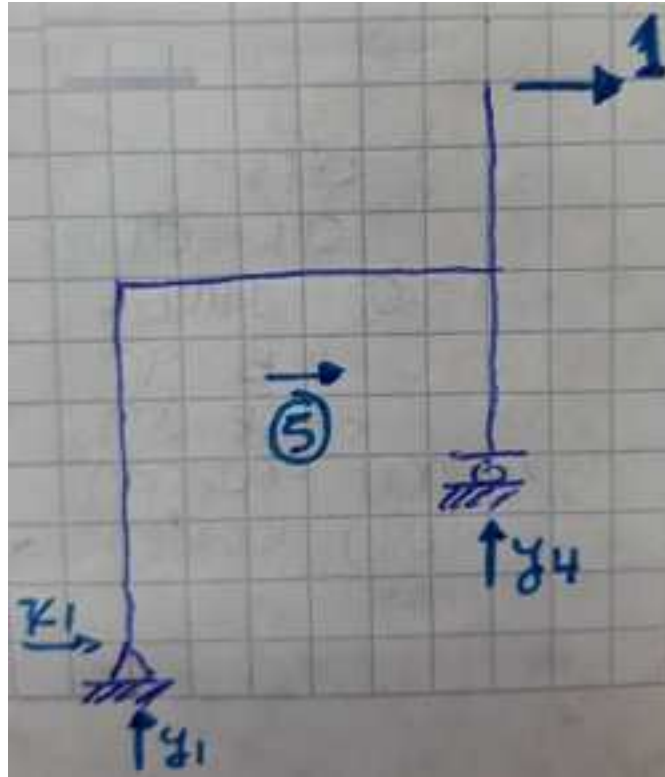
Funciones de momento flector:

$$[mf_{1n3}(x) :=, mf_{2n3}(x) :=, mf_{3n3}(x) :=, mf_{4n3}(x) :=]$$

$$\left[ \begin{array}{l} mf_{1n3}(x) := -x_{1n3} \cdot x \\ mf_{2n3}(x) := -m_{23n3} + y_{23n3} \cdot x \\ mf_{3n3}(x) := 0 \\ mf_{4n3}(x) := -m_{35n3} - x_{35n3} \cdot x \end{array} \right]$$

$$\left[ \begin{array}{l} mf_{1n3}(x) := x \\ mf_{2n3}(x) := 4 - 0.8 \cdot x \\ mf_{3n3}(x) := 0 \\ mf_{4n3}(x) := 0 \end{array} \right]$$

Aplicación de una **carga unitaria ficticia en el nudo 5:**



Equilibrio estático:

$[x_{1n5} :=, y_{1n5} :=, y_{4n5} :=]$

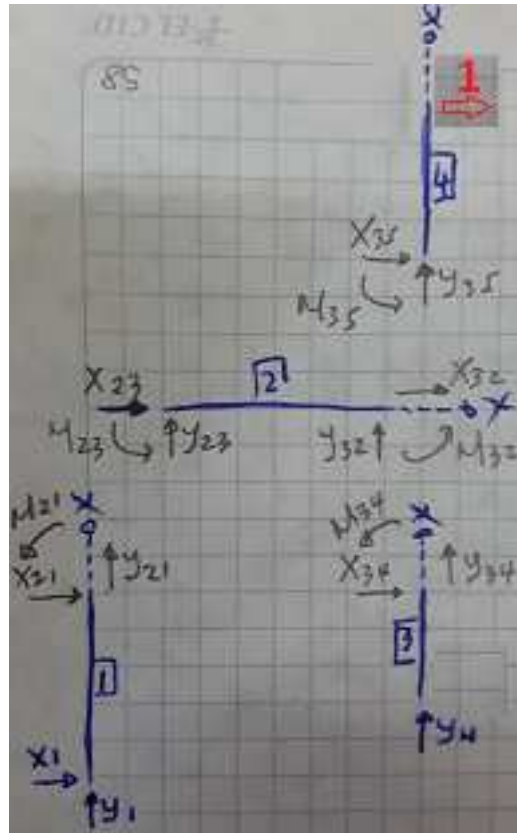
$$\begin{bmatrix} x_{1n5} + 1 = 0 \\ y_{1n5} + y_{4n5} = 0 \\ y_{4n5} \cdot 5 - 1.7 = 0 \end{bmatrix}$$

$$[x_{1n5} := -1, y_{1n5} := -1.4, y_{4n5} := 1.4]$$

Diagramas de cuerpo libre:

$$\begin{bmatrix} x_{21n5} := & y_{21n5} := & m_{21n5} := \\ x_{23n5} := & y_{23n5} := & m_{23n5} := \\ x_{32n5} := & y_{32n5} := & m_{32n5} := \\ x_{34n5} := & y_{34n5} := & m_{34n5} := \\ x_{35n5} := & y_{35n5} := & m_{35n5} := \end{bmatrix}$$





Secuencia: elemento e1, nudo n2, e3, e4, n3:

$$\left[ \begin{array}{ccc} x_{21n5} + x_{1n5} = 0 & y_{21n5} + y_{1n5} = 0 & m_{21n5} + x_{1n5} \cdot 4 = 0 \\ x_{21n5} + x_{23n5} = 0 & y_{21n5} + y_{23n5} = 0 & m_{21n5} + m_{23n5} = 0 \\ x_{34n5} = 0 & y_{4n5} + y_{34n5} = 0 & m_{34n5} = 0 \\ x_{35n5} + 1 = 0 & y_{35n5} = 0 & m_{35n5} - 1 \cdot 3 = 0 \\ x_{32n5} + x_{34n5} + x_{35n5} = 0 & y_{32n5} + y_{34n5} + y_{35n5} = 0 & m_{32n5} + m_{34n5} + m_{35n5} = 0 \end{array} \right]$$

$$\left[ \begin{array}{ccccc} m_{21n5} := 4 & m_{23n5} := -4 & m_{32n5} := -3 & m_{34n5} := 0 & m_{35n5} := 3 \\ x_{21n5} := 1 & x_{23n5} := -1 & x_{32n5} := 1 & x_{34n5} := 0 & x_{35n5} := -1 \\ y_{21n5} := 1.4 & y_{23n5} := -1.4 & y_{32n5} := 1.4 & y_{34n5} := -1.4 & y_{35n5} := 0 \end{array} \right]$$

Chequeo en elemento 2:

$$[x_{23n5} + x_{32n5} = 0, y_{23n5} + y_{32n5} = 0, m_{23n5} + m_{32n5} + y_{32n5} \cdot 5 = 0]$$

$$[true, true, true]$$

Funciones de momento flector:

$$[mf_{1n5}(x) :=, mf_{2n5}(x) :=, mf_{3n5}(x) :=, mf_{4n5}(x) :=]$$

$$\begin{bmatrix} mf1n5(x) := -x1n5 \cdot x \\ mf2n5(x) := -m23n5 + y23n5 \cdot x \\ mf3n5(x) := 0 \\ mf4n5(x) := -m35n5 - x35n5 \cdot x \end{bmatrix}$$

$$\begin{bmatrix} mf1n5(x) := x \\ mf2n5(x) := 4 - 1.4 \cdot x \\ mf3n5(x) := 0 \\ mf4n5(x) := x - 3 \end{bmatrix}$$

**Deformaciones laterales (sentido +X) en los nudos 3,4 y 5:**

[dx3 :=, dx4 :=, dx5 :=]

$$\begin{bmatrix} dx4 = \frac{1}{E} \cdot \left( \frac{1}{Ico1} \cdot \left( \int_0^4 MF1(x) \cdot mf1n4(x) dx + \int_0^3 MF3(x) \cdot mf3n4(x) dx + \int_0^3 MF4(x) \cdot mf4n4(x) \right. \right. \\ dx3 = \frac{1}{E} \cdot \left( \frac{1}{Ico1} \cdot \left( \int_0^4 MF1(x) \cdot mf1n3(x) dx + \int_0^3 MF3(x) \cdot mf3n3(x) dx + \int_0^3 MF4(x) \cdot mf4n3(x) \right. \right. \\ dx5 = \frac{1}{E} \cdot \left( \frac{1}{Ico1} \cdot \left( \int_0^4 MF1(x) \cdot mf1n5(x) dx + \int_0^3 MF3(x) \cdot mf3n5(x) dx + \int_0^3 MF4(x) \cdot mf4n5(x) \right. \right. \end{bmatrix}$$

$$\left. \begin{matrix} dx \left) + \frac{1}{Ivig} \cdot \int_0^5 MF2(x) \cdot mf2n4(x) dx \right) \\ dx \left) + \frac{1}{Ivig} \cdot \int_0^5 MF2(x) \cdot mf2n3(x) dx \right) \\ dx \left) + \frac{1}{Ivig} \cdot \int_0^5 MF2(x) \cdot mf2n5(x) dx \right) \end{matrix} \right]$$

$$dx4 = \frac{1}{E} \cdot \left( \frac{1}{Ico1} \cdot \left( \int_0^4 (70 \cdot x) \cdot x dx + \int_0^3 0 \cdot (-x) dx + \int_0^3 (30 \cdot x - 90) \cdot 0 dx \right) + \frac{1}{Ivig} \cdot \int_0^5 (280 - 74 \cdot x) \cdot (4 - 0.2 \cdot x) dx \right)$$

$$dx3 = \frac{1}{E} \cdot \left( \frac{1}{I_{col}} \cdot \left( \int_0^4 (70 \cdot x) \cdot x \, dx + \int_0^3 0 \cdot 0 \, dx + \int_0^3 (30 \cdot x - 90) \cdot 0 \, dx \right) + \frac{1}{I_{vig}} \cdot \int_0^5 (280 - 74 \cdot x) \cdot (4 - 0.8 \cdot x) \, dx \right)$$

$$dx5 = \frac{1}{E} \cdot \left( \frac{1}{I_{col}} \cdot \left( \int_0^4 (70 \cdot x) \cdot x \, dx + \int_0^3 0 \cdot 0 \, dx + \int_0^3 (30 \cdot x - 90) \cdot (x - 3) \, dx \right) + \frac{1}{I_{vig}} \cdot \int_0^5 (280 - 74 \cdot x) \cdot (4 - 1.4 \cdot x) \, dx \right)$$

$$(280 - 74 \cdot x) \cdot (4 - 1.4 \cdot x) \, dx$$

$$\begin{bmatrix} dx4 = 0.1614932392 \\ dx3 = 0.1438565549 \\ dx5 = 0.1322466563 \end{bmatrix}$$

Abajo se muestra la deformación correspondiente al análisis bajo el método de los Elementos finitos:

