



#1: [CaseMode := Sensitive, InputMode := Word]

Fórmula para rotación de ejes:

Coordenada (x,y) en el sistema x',y':

#2:
$$\begin{bmatrix} x_p = x \cdot \cos(\theta) + y \cdot \sin(\theta) \\ y_p = y \cdot \cos(\theta) - x \cdot \sin(\theta) \end{bmatrix}$$

Inercia respecto a los ejes x e y:

#3:
$$\begin{bmatrix} I_x = \int y^2 dA \\ I_y = \int x^2 dA \end{bmatrix}$$

Inercia respecto a los ejes del sistema x' e y':

#4:
$$\begin{bmatrix} I_{x_p} = \int y_p^2 dA \\ I_{y_p} = \int x_p^2 dA \end{bmatrix}$$

Conversión:

#5:
$$\begin{bmatrix} I_{x_p} = \int (y \cdot \cos(\theta) - x \cdot \sin(\theta))^2 dA \\ I_{y_p} = \int (x \cdot \cos(\theta) + y \cdot \sin(\theta))^2 dA \end{bmatrix}$$

#6:
$$\begin{bmatrix} I_{x_p} = \int (y^2 \cdot \cos^2(\theta) - 2 \cdot y \cdot \cos(\theta) \cdot x \cdot \sin(\theta) + x^2 \cdot \sin^2(\theta)) dA \\ I_{y_p} = \int (x^2 \cdot \cos^2(\theta) + 2 \cdot x \cdot \cos(\theta) \cdot y \cdot \sin(\theta) + y^2 \cdot \sin^2(\theta)) dA \end{bmatrix}$$

$$\#7: \begin{bmatrix} I_{xp} = \cos^2(\theta) \cdot \int y^2 dA - 2 \cdot \cos(\theta) \cdot \sin(\theta) \cdot \int x \cdot y dA + \sin^2(\theta) \cdot \int x^2 dA \\ I_{yp} = \cos^2(\theta) \cdot \int x^2 dA + 2 \cdot \cos(\theta) \cdot \sin(\theta) \cdot \int x \cdot y dA + \sin^2(\theta) \cdot \int y^2 dA \end{bmatrix}$$

$$\#8: \begin{bmatrix} I_{xp} = I_x \cdot \cos^2(\theta) - 2 \cdot I_{xy} \cdot \sin(\theta) \cdot \cos(\theta) + I_y \cdot \sin^2(\theta) \\ I_{yp} = I_y \cdot \cos^2(\theta) + 2 \cdot I_{xy} \cdot \sin(\theta) \cdot \cos(\theta) + I_x \cdot \sin^2(\theta) \end{bmatrix}$$

Identidades trigonométricas:

$$\#9: \begin{bmatrix} \cos^2(\theta) = \frac{1 + \cos(2 \cdot \theta)}{2} \quad \sin^2(\theta) = \frac{1 - \cos(2 \cdot \theta)}{2} \\ \sin(2 \cdot \theta) = 2 \cdot \sin(\theta) \cdot \cos(\theta) \quad \cos(2 \cdot \theta) = \cos^2(\theta) - \sin^2(\theta) \end{bmatrix}$$

$$\#10: \begin{bmatrix} I_{xp} = I_x \cdot \frac{1 + \cos(2 \cdot \theta)}{2} - I_{xy} \cdot \sin(2 \cdot \theta) + I_y \cdot \frac{1 - \cos(2 \cdot \theta)}{2} \\ I_{yp} = I_y \cdot \frac{1 + \cos(2 \cdot \theta)}{2} + I_{xy} \cdot \sin(2 \cdot \theta) + I_x \cdot \frac{1 - \cos(2 \cdot \theta)}{2} \end{bmatrix}$$

Usar estas fórmulas:

$$\#11: \begin{bmatrix} I_{xp} = \frac{I_x - I_y}{2} \cdot \cos(2 \cdot \theta) - I_{xy} \cdot \sin(2 \cdot \theta) + \frac{I_x + I_y}{2} \\ I_{yp} = \frac{I_y - I_x}{2} \cdot \cos(2 \cdot \theta) + I_{xy} \cdot \sin(2 \cdot \theta) + \frac{I_x + I_y}{2} \end{bmatrix}$$

Buscar el ángulo θ que genere las mayores y las menores inercias I_{xp} e I_{yp} :

Se ajusta la forma de la ecuación:

$$\#12: \begin{bmatrix} I_{xp} = \frac{I_x - I_y}{2} \cdot \cos(\theta^2) - I_{xy} \cdot \sin(\theta^2) + \frac{I_x + I_y}{2} \\ I_{yp} = \frac{I_y - I_x}{2} \cdot \cos(\theta^2) + I_{xy} \cdot \sin(\theta^2) + \frac{I_x + I_y}{2} \end{bmatrix}$$

$$\#13: \begin{bmatrix} \frac{d}{d \theta^2} \left(\frac{I_x - I_y}{2} \cdot \cos(\theta^2) - I_{xy} \cdot \sin(\theta^2) + \frac{I_x + I_y}{2} \right) = 0 \\ \frac{d}{d \theta^2} \left(\frac{I_y - I_x}{2} \cdot \cos(\theta^2) + I_{xy} \cdot \sin(\theta^2) + \frac{I_x + I_y}{2} \right) = 0 \end{bmatrix}$$

$$\#14: \begin{bmatrix} I_{xy} \cdot \cos(\theta^2) + \frac{(I_x - I_y) \cdot \sin(\theta^2)}{2} = 0 \\ I_{xy} \cdot \cos(\theta^2) + \frac{(I_x - I_y) \cdot \sin(\theta^2)}{2} = 0 \end{bmatrix}$$

$$\#15: \text{SOLVE} \left(\begin{bmatrix} I_{xy} \cdot \cos(\theta^2) + \frac{(I_x - I_y) \cdot \sin(\theta^2)}{2} = 0 \\ I_{xy} \cdot \cos(\theta^2) + \frac{(I_x - I_y) \cdot \sin(\theta^2)}{2} = 0 \end{bmatrix}, \theta^2 \right)$$

$$\#16: \left[\theta_2 = - \operatorname{ATAN}\left(\frac{2 \cdot I_{xy}}{I_x - I_y}\right), \theta_2 = \pi - \operatorname{ATAN}\left(\frac{2 \cdot I_{xy}}{I_x - I_y}\right), \theta_2 = - \operatorname{ATAN}\left(\frac{2 \cdot I_{xy}}{I_x - I_y}\right) - \pi \right]$$

$$\#17: \left[2 \cdot \theta = - \operatorname{ATAN}\left(\frac{2 \cdot I_{xy}}{I_x - I_y}\right), 2 \cdot \theta = \pi - \operatorname{ATAN}\left(\frac{2 \cdot I_{xy}}{I_x - I_y}\right), 2 \cdot \theta = - \operatorname{ATAN}\left(\frac{2 \cdot I_{xy}}{I_x - I_y}\right) - \pi \right]$$

$$\#18: \left[2 \cdot \theta + 0 = - \operatorname{ATAN}\left(\frac{2 \cdot I_{xy}}{I_x - I_y}\right), 2 \cdot \theta - \pi = - \operatorname{ATAN}\left(\frac{2 \cdot I_{xy}}{I_x - I_y}\right), 2 \cdot \theta + \pi = - \operatorname{ATAN}\left(\frac{2 \cdot I_{xy}}{I_x - I_y}\right) \right]$$

$$\#19: \left[\operatorname{TAN}(2 \cdot \theta + 0) = \operatorname{TAN}\left(- \operatorname{ATAN}\left(\frac{2 \cdot I_{xy}}{I_x - I_y}\right)\right), \operatorname{TAN}(2 \cdot \theta - \pi) = \operatorname{TAN}\left(- \operatorname{ATAN}\left(\frac{2 \cdot I_{xy}}{I_x - I_y}\right)\right), \operatorname{TAN}(2 \cdot \theta + \pi) = \right. \\ \left. \operatorname{TAN}\left(- \operatorname{ATAN}\left(\frac{2 \cdot I_{xy}}{I_x - I_y}\right)\right) \right]$$

$$\#20: \left[\operatorname{TAN}(2 \cdot \theta) = \frac{2 \cdot I_{xy}}{I_y - I_x}, \operatorname{TAN}(2 \cdot \theta) = \frac{2 \cdot I_{xy}}{I_y - I_x}, \operatorname{TAN}(2 \cdot \theta) = \frac{2 \cdot I_{xy}}{I_y - I_x} \right]$$

Ángulo para el cual las inercias serán mayores y menores:

$$\#21: \operatorname{TAN}(2 \cdot \theta) = \frac{2 \cdot I_{xy}}{I_y - I_x}$$