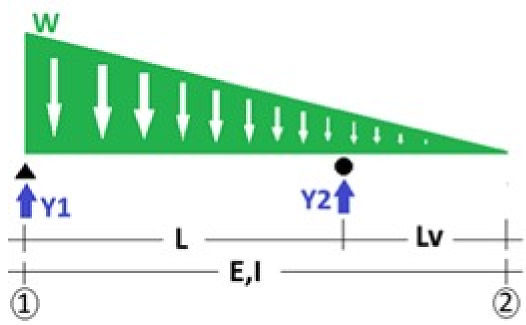


#1: [CaseMode ≔ Sensitive, InputMode ≔ Word]

#2: [Y1 ≔, Y2 ≔, W ≔, L ≔, Lv ≔, E ≔, I ≔]

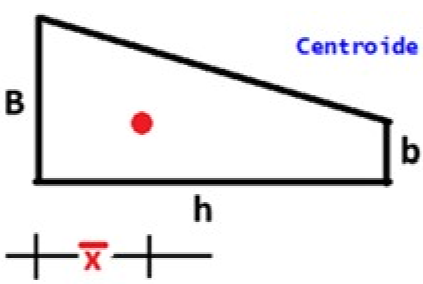
#3: [MF1(x) ≔, MF2(x) ≔, w(x) ≔, δ1(x) ≔, δ2(x) ≔, θ1 ≔, θ2 ≔, θ12 ≔, H(x) ≔]

Ecuaciones de equilibrio:

#4:
$$\left[\begin{array}{c} Y1 + Y2 - \dfrac{W\cdot(L + Lv)}{2} = 0 \\ Y2\cdot L - \dfrac{W\cdot(L + Lv)}{2}\cdot\dfrac{1}{3}\cdot(L + Lv) = 0 \end{array}\right]$$

#5:
$$\left[Y1 := \frac{W\cdot(L + Lv)\cdot(2\cdot L - Lv)}{6\cdot L},\ Y2 := \frac{W\cdot(L + Lv)^2}{6\cdot L}\right]$$

Para el trapecio de carga se usa el valor de la carga y el centroide de un trapecio, pero puede usarse dos cargas, una triangular y otra rectangular



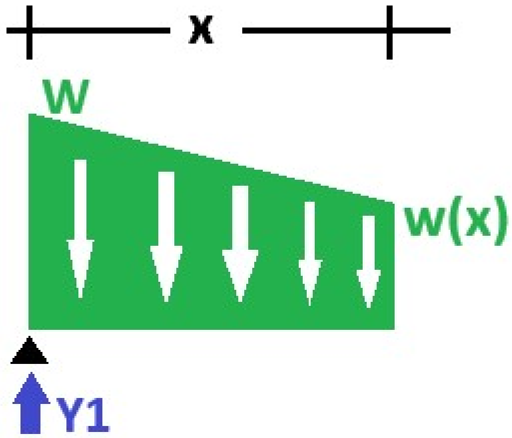
Centroide de una figura trapezoidal medido desde la base B:

$$\frac{(B - b)\cdot h}{2}\cdot\frac{h}{3} + b\cdot h\cdot\frac{h}{2} = \frac{B + b}{2}\cdot h\cdot cg$$

$$\text{SOLVE}\left(\frac{(B - b)\cdot h}{2}\cdot\frac{h}{3} + b\cdot h\cdot\frac{h}{2} = \frac{B + b}{2}\cdot h\cdot cg,\ cg\right)$$

$$cg = \frac{h\cdot(B + 2\cdot b)}{3\cdot(B + b)} \lor h = 0$$

Valor de la carga en un punto x:

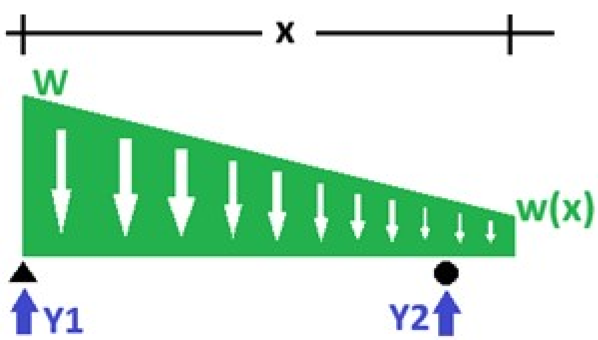


#6: $$w(x) := \frac{W}{L + Lv}\cdot(L + Lv - x)$$

Ecuaciones del diagrama de momento flector:

#7: $$MF1(x) := Y1\cdot x - \frac{W + w(x)}{2}\cdot x\cdot\left(x - \frac{x\cdot(W + 2\cdot w(x))}{3\cdot(W + w(x))}\right)$$

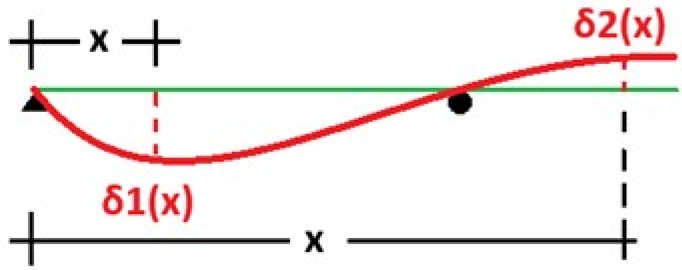
#8: $$MF1(x) := -\frac{x\cdot(L\cdot x\cdot w(x) + W\cdot(2\cdot L\cdot x - (L + Lv)\cdot(2\cdot L - Lv)))}{6\cdot L}$$



#9: $$MF2(x) := \left(Y1\cdot x - \frac{W + w(x)}{2}\cdot x\cdot\left(x - \frac{x\cdot(W + 2\cdot w(x))}{3\cdot(W + w(x))}\right)\right) + Y2\cdot(x - L)$$

#10: $$MF2(x) := \frac{W\cdot(x^3 - 3\cdot x^2\cdot(L + Lv) + 3\cdot x\cdot(L + Lv)^2 - (L + Lv)^3)}{6\cdot(L + Lv)}$$

Curva elástica estimada (sin escala):



δ12: Distancia entre la tangente en el punto 1 de la curva elástica al punto 2 de la curva elástica:

#11: $$\delta 12 := \frac{1}{E \cdot I} \cdot \int_0^L MF1(x) \cdot (L - x)\, dx$$

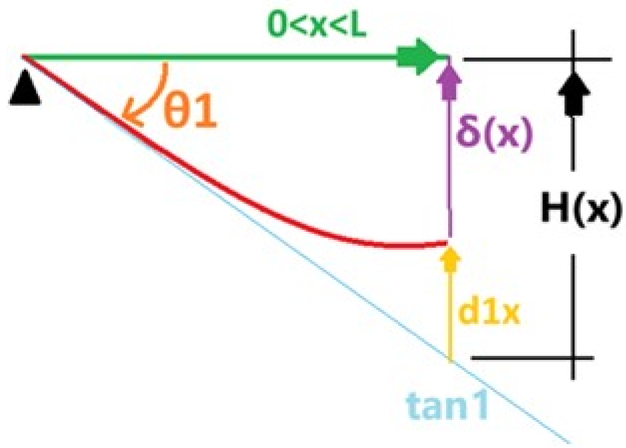
#12: $$\delta 12 := \frac{L^2 \cdot W \cdot (8 \cdot L^3 + 15 \cdot L^2 \cdot Lv - 10 \cdot Lv^3)}{360 \cdot E \cdot I \cdot (L + Lv)}$$

Giro del punto 1 (estamos en el campo de pequeñas deformaciones, donde TAN (θ)=θ):

#13: $$\theta 1 := - \frac{\delta 12}{L}$$

#14: $$\theta 1 := - \frac{L \cdot W \cdot (8 \cdot L^3 + 15 \cdot L^2 \cdot Lv - 10 \cdot Lv^3)}{360 \cdot E \cdot I \cdot (L + Lv)}$$

Deformación por desplazamiento en un punto 0<x<L:



#15: $H(x) :\!= \theta 1 \cdot x$

#16: $$H(x) :\!= - \frac{L \cdot W \cdot x \cdot (8 \cdot L^3 + 15 \cdot L^2 \cdot Lv - 10 \cdot Lv^3)}{360 \cdot E \cdot I \cdot (L + Lv)}$$

Se usa temporalmente la variable J para referirse a la variable x

#17: $$d1J = \frac{1}{E \cdot I} \cdot \int_0^J MF1(x) \cdot (J - x)\, dx$$

#18: $$d1J = \frac{J^3 \cdot W \cdot (3 \cdot J^2 \cdot L - 15 \cdot J \cdot L \cdot (L + Lv) + 10 \cdot (L + Lv)^2 \cdot (2 \cdot L - Lv))}{360 \cdot E \cdot I \cdot L \cdot (L + Lv)}$$

#19: $$\text{SUBST}\left(d1J = \frac{J^3 \cdot W \cdot (3 \cdot J^2 \cdot L - 15 \cdot J \cdot L \cdot (L + Lv) + 10 \cdot (L + Lv)^2 \cdot (2 \cdot L - Lv))}{360 \cdot E \cdot I \cdot L \cdot (L + Lv)}, [d1J, J], [d1x, x]\right)$$

#20: $$d1x :\!= \frac{W \cdot x^3 \cdot (3 \cdot L \cdot x^2 - 15 \cdot L \cdot x \cdot (L + Lv) + 10 \cdot (L + Lv)^2 \cdot (2 \cdot L - Lv))}{360 \cdot E \cdot I \cdot L \cdot (L + Lv)}$$

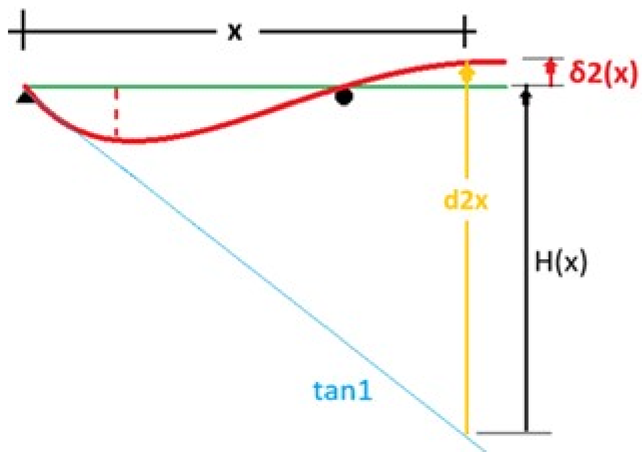
#21: $\delta1(x) :\!= H(x) + d1x$

#22: $\delta1(x) :\!=$

$$\frac{W\cdot x\cdot(3\cdot L\cdot x^4 - 15\cdot L\cdot x^3\cdot(L + Lv) + 10\cdot x^2\cdot(L + Lv)^2\cdot(2\cdot L - Lv) - L^2\cdot(\sim}{360\cdot E\cdot I\cdot L\cdot(L + Lv)}\sim$$

$$\frac{8\cdot L^3 + 15\cdot L^2\cdot Lv - 10\cdot Lv^3))}{}$$

Deformación por desplazamiento en un punto L<x<(L+Lv):



#23: $$d2J = \frac{1}{E\cdot I}\cdot\left(\int_0^L MF1(x)\cdot(J - x)\,dx + \int_L^J MF2(x)\cdot(J - x)\,dx\right)$$

#24: $d2J =$

$$\frac{W\cdot(3\cdot J^5 - 15\cdot J^4\cdot(L + Lv) + 30\cdot J^3\cdot(L + Lv)^2 - 30\cdot J^2\cdot(L + Lv)^3 + 30\cdot\sim}{360\cdot E\cdot I\cdot(L + Lv)}\sim$$

$$\frac{J\cdot L\cdot(L + Lv)^3 - 10\cdot L^2\cdot(L + Lv)^3)}{}$$

#25: $$\mathrm{SUBST}\left(d2J = \frac{W\cdot(3\cdot J^5 - 15\cdot J^4\cdot(L + Lv) + 30\cdot J^3\cdot(L + Lv)^2 - 30\cdot J^2\cdot(L + Lv)^3 + 30\cdot J\cdot L\cdot(L + Lv)^3 - 10\cdot L^2\cdot(L + Lv)^3)}{360\cdot E\cdot I\cdot(L + Lv)}, [d2J, J], [d2x, x]\right)$$

#26: $$d2x = \frac{W\cdot(3\cdot x^5 - 15\cdot x^4\cdot(L + Lv) + 30\cdot x^3\cdot(L + Lv)^2 - 30\cdot x^2\cdot(L + Lv)^3 + 30\cdot L\cdot x\cdot(L + Lv)^3 - 10\cdot L^2\cdot(L + Lv)^3)}{360\cdot E\cdot I\cdot(L + Lv)}$$

#27: $\delta 2(x) :\!= d2x + H(x)$

#28: $$\delta 2(x) :\!= \frac{W\cdot(3\cdot x^5 - 15\cdot x^4\cdot(L + Lv) + 30\cdot x^3\cdot(L + Lv)^2 - 30\cdot x^2\cdot(L + Lv)^3 + L\cdot x\cdot(22\cdot L^3 + 75\cdot L^2\cdot Lv + 90\cdot L\cdot Lv^2 + 40\cdot Lv^3) - 10\cdot L^2\cdot(L + Lv)^3)}{360\cdot E\cdot I\cdot(L + Lv)}$$

Ejercicio para valores numéricos con L=10,Lv=5,W=8:

#29: [L :=, Lv :=, W :=]

#30: [L := 10, Lv := 5, W := 8]

#31: $[Y1 := 30, Y2 := 30]$

#32: $$\begin{bmatrix} MF1(x) \\ MF2(x) \end{bmatrix} = \frac{1}{45}\cdot\begin{bmatrix} 2\cdot x\cdot(2\cdot x^2 - 90\cdot x + 675) \\ 4\cdot(x^3 - 45\cdot x^2 + 675\cdot x - 3375) \end{bmatrix}$$

#33: $$\begin{bmatrix} MF1(x) \\ MF2(x) \end{bmatrix} = \begin{bmatrix} \frac{4\cdot x^3}{45} - 4\cdot x^2 + 30\cdot x \\ \frac{4\cdot x^3}{45} - 4\cdot x^2 + 60\cdot x - 300 \end{bmatrix}$$

#34: $$\begin{bmatrix} MF1(x) \\ MF2(x) \end{bmatrix} = \begin{bmatrix} 0.08888888888\cdot x^3 - 4\cdot x^2 + 30\cdot x \\ 0.08888888888\cdot x^3 - 4\cdot x^2 + 60\cdot x - 300 \end{bmatrix}$$

#35: $$\begin{bmatrix} \delta12 \\ \delta12 \end{bmatrix} = \frac{1}{E\cdot I}\cdot\begin{bmatrix} \frac{19000}{9} \\ 2111.111111 \end{bmatrix}$$

#36: $$\begin{bmatrix} \theta1 \\ \theta1 \end{bmatrix} = \frac{1}{E\cdot I}\cdot\begin{bmatrix} -\frac{1900}{9} \\ -211.1111111 \end{bmatrix}$$

#37: $$\begin{bmatrix} H(x) \\ H(x) \end{bmatrix} = \frac{1}{E\cdot I}\cdot\begin{bmatrix} -\frac{1900\cdot x}{9} \\ -211.1111111\cdot x \end{bmatrix}$$

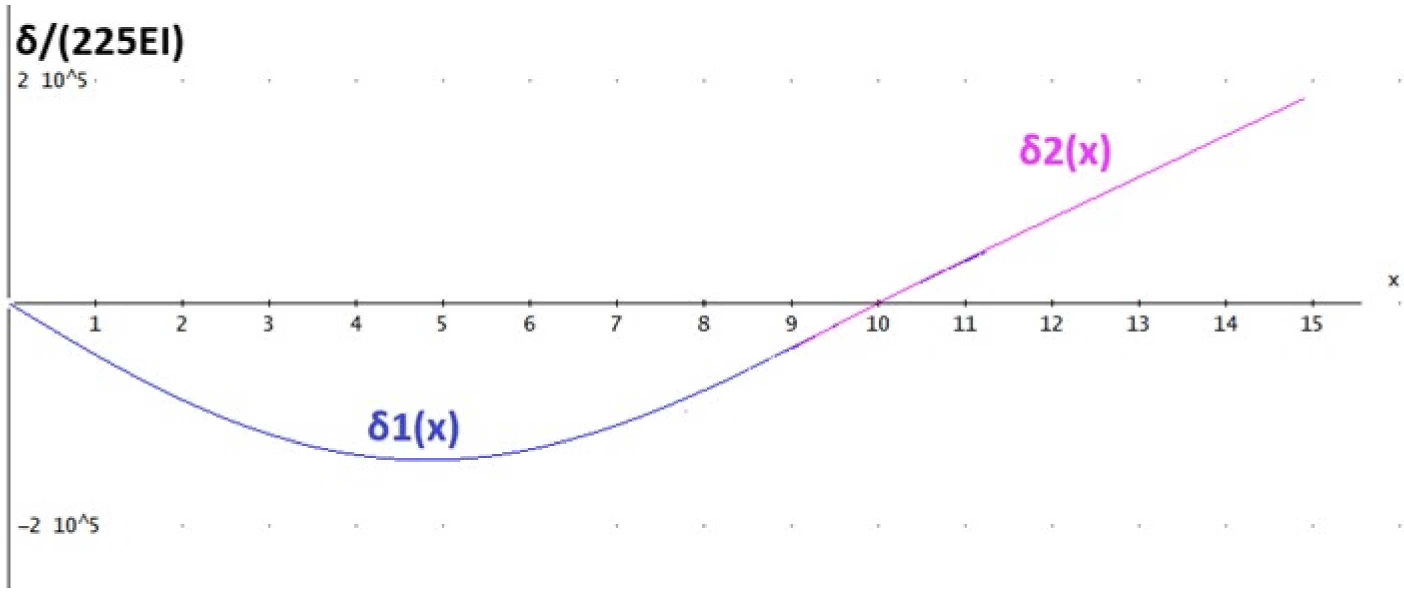
#38: $$\begin{bmatrix} d1x \\ d2x \end{bmatrix} = \frac{1}{225\cdot E\cdot I}\cdot\begin{bmatrix} x^3\cdot(x^2 - 75\cdot x + 1125) \\ x^5 - 75\cdot x^4 + 2250\cdot x^3 - 33750\cdot x^2 + 337500\cdot x - 1125000 \end{bmatrix}$$

#39: $$\begin{bmatrix} d1x \\ d2x \end{bmatrix} = \frac{1}{E \cdot I} \cdot \begin{bmatrix} \dfrac{x^5}{225} - \dfrac{x^4}{3} + 5 \cdot x^3 \\ \dfrac{x^5}{225} - \dfrac{x^4}{3} + 10 \cdot x^3 - 150 \cdot x^2 + 1500 \cdot x - 5000 \end{bmatrix}$$

#40: $$\begin{bmatrix} \delta 1(x) \\ \delta 2(x) \end{bmatrix} = \frac{1}{225 \cdot E \cdot I} \cdot \begin{bmatrix} x \cdot (x^4 - 75 \cdot x^3 + 1125 \cdot x^2 - 47500) \\ x^5 - 75 \cdot x^4 + 2250 \cdot x^3 - 33750 \cdot x^2 + 290000 \cdot x - 1125000 \end{bmatrix}$$

#41: $$\begin{bmatrix} \delta 1(x) \\ \delta 2(x) \end{bmatrix} = \frac{1}{E \cdot I} \cdot \begin{bmatrix} \dfrac{x^5}{225} - \dfrac{x^4}{3} + 5 \cdot x^3 - \dfrac{1900 \cdot x}{9} \\ \dfrac{x^5}{225} - \dfrac{x^4}{3} + 10 \cdot x^3 - 150 \cdot x^2 + \dfrac{11600 \cdot x}{9} - 5000 \end{bmatrix}$$

#42: $$\begin{bmatrix} \delta 1(x) \\ \delta 2(x) \end{bmatrix} = \frac{1}{E \cdot I} \cdot \begin{bmatrix} 0.004444444444 \cdot x^5 - 0.3333333333 \cdot x^4 + 5 \cdot x^3 - 211.1111111 \cdot x \\ 0.004444444444 \cdot x^5 - 0.3333333333 \cdot x^4 + 10 \cdot x^3 - 150 \cdot x^2 + 1288.888888 \cdot x - 5000 \end{bmatrix}$$



Deformación máxima en la luz:

#43: $\text{SOLVE}\left(\dfrac{d}{dx}\,(\delta 1(x) = 0),\ x\right)$

#44: $x = 13.30727816 \lor x = -3.280781303 \lor x = 45.15451438 \lor x = 4.818988753$

#45: $\delta 1(4.818988753)$

#46: $$-\frac{626.0070256}{E\cdot I}$$

Deformación en el voladizo:

#47: $\delta 2(L + Lv)$

#48: $$\frac{833.3333333}{E\cdot I}$$