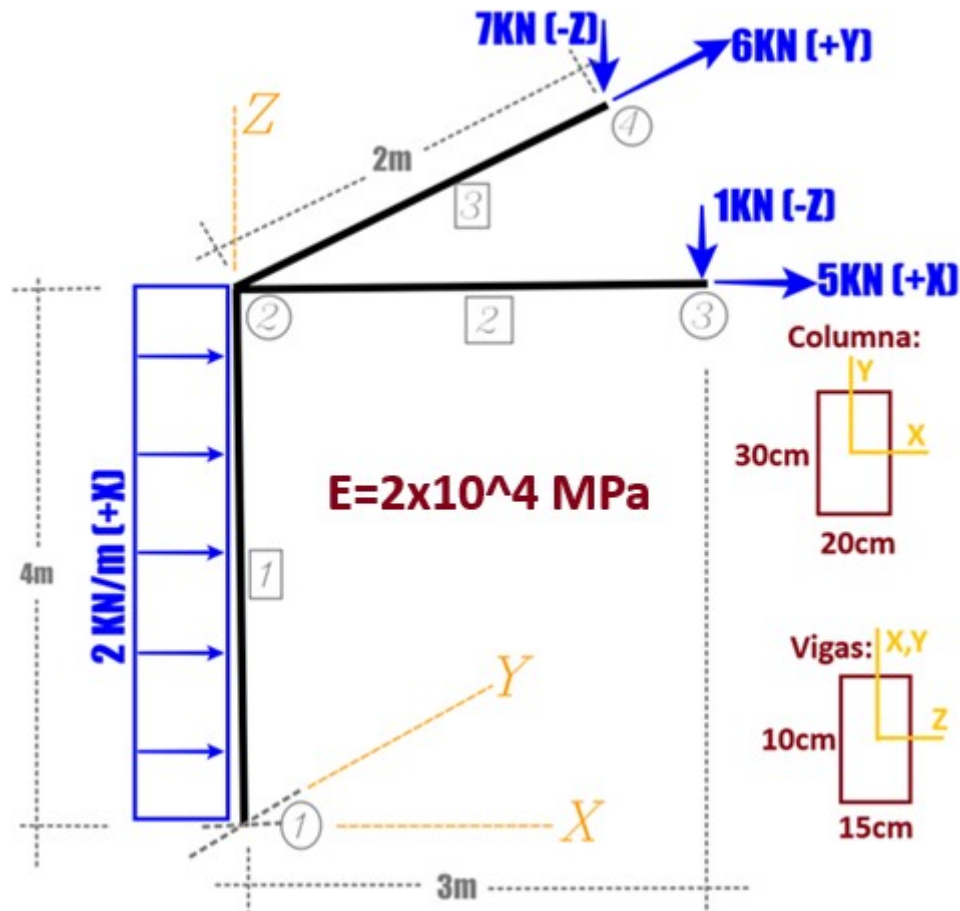


Con el método de la Carga Unitaria Ficticia, resuelva las deformaciones por desplazamiento de los nudos 3 y 4 de la estructura siguiente:



#1: [CaseMode := Sensitive, InputMode := Word]

Inercias y módulo de elasticidad:

#2:

$$\left[\begin{array}{cc} E := 2 \cdot 10^7 & 0 \\ I_{1x} := \frac{1}{12} \cdot (0.2 \cdot 0.3^3) & I_{1y} := \frac{1}{12} \cdot (0.3 \cdot 0.2^3) \\ I_{2z} := \frac{1}{12} \cdot (0.15 \cdot 0.1^3) & I_{2y} := \frac{1}{12} \cdot (0.1 \cdot 0.15^3) \\ I_{3z} := \frac{1}{12} \cdot (0.15 \cdot 0.1^3) & I_{3x} := \frac{1}{12} \cdot (0.1 \cdot 0.15^3) \end{array} \right]$$

#3:

$$\left[\begin{array}{cc} E := 20000000 & 0 \\ I1x := \frac{9}{20000} & I1y := \frac{1}{5000} \\ I2z := \frac{1}{80000} & I2y := \frac{9}{320000} \\ I3z := \frac{1}{80000} & I3x := \frac{9}{320000} \end{array} \right]$$

#4:

$$\left[\begin{array}{cc} E := 2 \cdot 10^7 & 0 \\ I1x := 0.00045 & I1y := 0.0002 \\ I2z := 1.25 \cdot 10^{-5} & I2y := 2.8125 \cdot 10^{-5} \\ I3z := 1.25 \cdot 10^{-5} & I3x := 2.8125 \cdot 10^{-5} \end{array} \right]$$

$$\#5: [E \cdot I1x, E \cdot I1y, E \cdot I2y, E \cdot I2z, E \cdot I3z, E \cdot I3x] = \left[9000, 4000, \frac{1125}{2}, 250, 250, \frac{1125}{2} \right]$$

$$\#6: \left[\frac{1}{E \cdot I1x}, \frac{1}{E \cdot I1y}, \frac{1}{E \cdot I2y}, \frac{1}{E \cdot I2z}, \frac{1}{E \cdot I3z}, \frac{1}{E \cdot I3x} \right] = \left[\frac{1}{9000}, \frac{1}{4000}, \frac{2}{1125}, \frac{1}{250}, \frac{1}{250}, \frac{2}{1125} \right]$$

Momentos flectores "minúscula" de la estructura con cargas unitarias ficticias en el nudo 4:

$$\#7: \left[\begin{array}{ccc} mf1x4(x) := & mf2x4(x) := & mf3x4(x) := \\ mf1y4(x) := & mf2y4(x) := & mf3y4(x) := \\ mf1z4(x) := & mf2z4(x) := & mf3z4(x) := \end{array} \right]$$

Cálculo de las reacciones del nudo 1 empotrado:
 $[\Sigma Fx=0; \Sigma Fy=0; \Sigma Fz=0; \Sigma Mx1=0; \Sigma My1=0; \Sigma Mz1=0]$

$$\#8: [X1 :=, Y1 :=, Z1 :=, Mx1 :=, My1 :=, Mz1 :=]$$

#9:

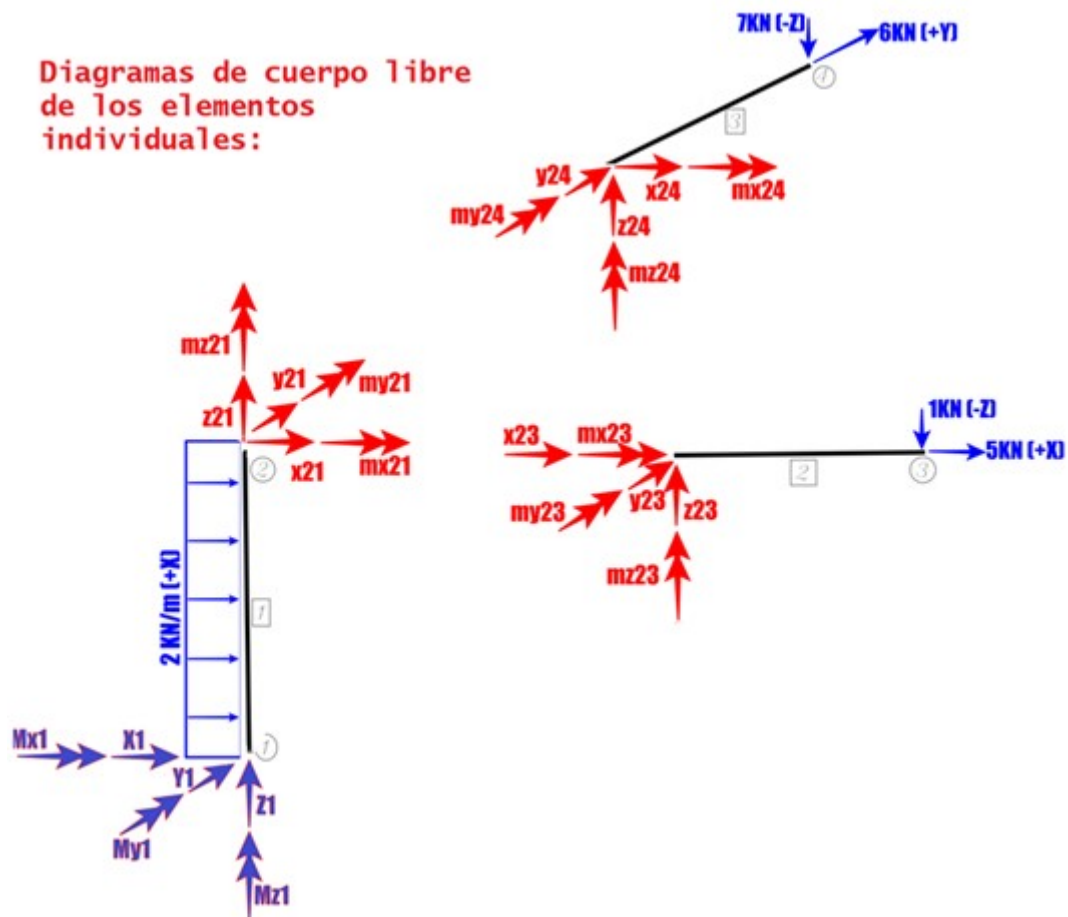
$$\begin{bmatrix} X1 + 2 \cdot 4 + 5 = 0 \\ Y1 + 6 = 0 \\ Z1 - 1 - 7 = 0 \\ Mx1 - 6 \cdot 4 - 7 \cdot 2 = 0 \\ My1 + 2 \cdot 4 \cdot 2 + 5 \cdot 4 + 1 \cdot 3 = 0 \\ Mz1 = 0 \end{bmatrix}$$

#10: $[Mx1 := 38, My1 := -39, Mz1 := 0, X1 := -13, Y1 := -6, Z1 := 8]$

Diagramas de cuerpo libre de los elementos individuales:

#11:

$$\begin{bmatrix} x21 := & y21 := & z21 := & x23 := & y23 := & z23 := & x24 := & y24 := & z24 := \\ mx21 := & my21 := & mz21 := & mx23 := & my23 := & mz23 := & mx24 := & my24 := & mz24 := \end{bmatrix}$$



$\sum F=0$ y $\sum M$ respecto a ejes en el nudo 2=0

Columna 1:

#12:

$$\begin{bmatrix} X1 + x21 + 2 \cdot 4 = 0 & Y1 + y21 = 0 & Z1 + z21 = 0 \\ Mx1 + mx21 + Y1 \cdot 4 = 0 & My1 + my21 - 2 \cdot 4 \cdot 2 - X1 \cdot 4 = 0 & Mz1 + mz21 = 0 \end{bmatrix}$$

#13: $[mx21 := -14, my21 := 3, mz21 := 0, x21 := 5, y21 := 6, z21 := -8]$

Viga 2:

#14:
$$\begin{bmatrix} x23 + 5 = 0 & y23 = 0 & z23 - 1 = 0 \\ mx23 = 0 & my23 + 1 \cdot 3 = 0 & mz23 = 0 \end{bmatrix}$$

#15: $[mx23 := 0, my23 := -3, mz23 := 0, x23 := -5, y23 := 0, z23 := 1]$

Viga 3:

#16:
$$\begin{bmatrix} x24 = 0 & y24 + 6 = 0 & z24 - 7 = 0 \\ mx24 - 7 \cdot 2 & my24 = 0 & mz24 = 0 \end{bmatrix}$$

#17: $[mx24 := 14, my24 := 0, mz24 := 0, x24 := 0, y24 := -6, z24 := 7]$

Chequeo de equilibrio en el nudo 2:

$\sum F_{internas} = \sum F_{externas}$

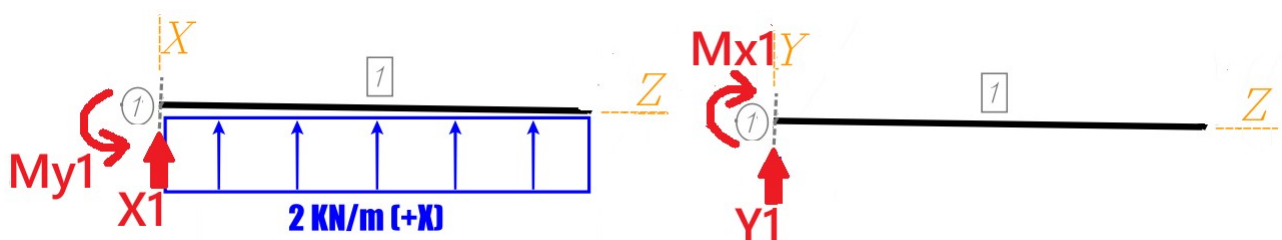
#18:
$$\begin{bmatrix} x21 + x23 + x24 = 0 & y21 + y23 + y24 = 0 & z21 + z23 + z24 = 0 \\ mx21 + mx23 + mx24 = 0 & my21 + my23 + my24 = 0 & mz21 + mz23 + mz24 = 0 \end{bmatrix}$$

#19:
$$\begin{bmatrix} \text{true} & \text{true} & \text{true} \\ \text{true} & \text{true} & \text{true} \end{bmatrix}$$

Calculo de los momentos flectores "mayúscula" de la estructura con la carga original:

#20:
$$\begin{bmatrix} MF1x(z) := MF1y(z) := \\ MF2y(x) := MF2z(x) := \\ MF3x(y) := MF3z(y) := \end{bmatrix}$$

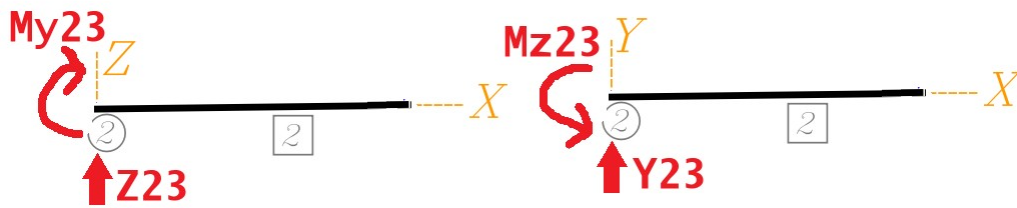
Columna 1:



#21:
$$\left[MF1y(z) := -My1 + X1 \cdot z + 2 \cdot z \cdot \frac{z}{2}, MF1x(z) := Mx1 + Y1 \cdot z \right]$$

#22:
$$\left[MF1y(z) := z^2 - 13 \cdot z + 39, MF1x(z) := 38 - 6 \cdot z \right]$$

Viga 2:



#23: $[MF2y(x) := my23 + z23 \cdot x, MF2z(x) := -mz23 + y23 \cdot x]$

#24: $[MF2y(x) := x - 3, MF2z(x) := 0]$

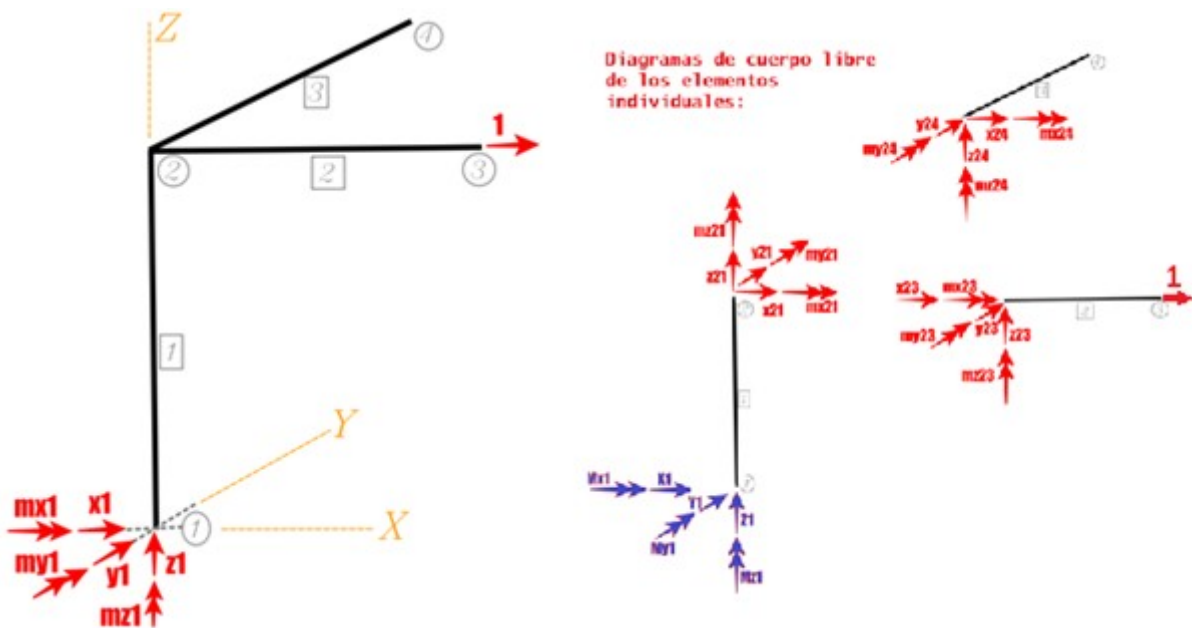
Viga 3:



#25: $[MF3x(y) := -mx24 + z24 \cdot y, MF3z(y) := mz24 + x24 \cdot y]$

#26: $[MF3x(y) := 7 \cdot y - 14, MF3z(y) := 0]$

Deformación en X del nudo 3 con el método de la Carga Unitaria Ficticia:



#27: $[x1 :=, y1 :=, z1 :=, mx1 :=, my1 :=, mz1 :=]$

#28:
$$\begin{bmatrix} x21 := & y21 := & z21 := & x23 := & y23 := & z23 := & x24 := & y24 := & z24 := \\ mx21 := & my21 := & mz21 := & mx23 := & my23 := & mz23 := & mx24 := & my24 := & mz24 := \end{bmatrix}$$

$$\#29: \begin{bmatrix} x1 + 1 = 0 \\ y1 = 0 \\ z1 = 0 \\ mx1 = 0 \\ my1 + 1 \cdot 4 = 0 \\ mz1 = 0 \end{bmatrix}$$

$$\#30: [mx1 := 0, my1 := -4, mz1 := 0, x1 := -1, y1 := 0, z1 := 0]$$

Columna 1:

$$\#31: \begin{bmatrix} x1 + x21 = 0 & y1 + y21 = 0 & z1 + z21 = 0 \\ mx1 + mx21 + y1 \cdot 4 & my1 + my21 - x1 \cdot 4 = 0 & mz1 + mz21 = 0 \end{bmatrix}$$

$$\#32: [mx21 := 0, my21 := 0, mz21 := 0, x21 := 1, y21 := 0, z21 := 0]$$

Viga 2:

$$\#33: \begin{bmatrix} x23 + 1 = 0 & y23 = 0 & z23 = 0 \\ mx23 = 0 & my23 = 0 & mz23 = 0 \end{bmatrix}$$

$$\#34: [mx23 := 0, my23 := 0, mz23 := 0, x23 := -1, y23 := 0, z23 := 0]$$

Viga 3=0

Momentos flectores "minúscula" de la estructura con cargas unitarias ficticias para desplazamiento en el nudo 3:

$$\#35: \begin{bmatrix} mf1xd3(z) := mx1 + y1 \cdot z & mf2yd3(x) := my23 + z23 \cdot x & mf3xd3(y) := 0 \\ mf1yd3(z) := -my1 + x1 \cdot z & mf2zd3(x) := -mz23 + y23 \cdot x & mf3zd3(y) := 0 \end{bmatrix}$$

$$\#36: \begin{bmatrix} mf1xd3(z) := 0 & mf2yd3(x) := 0 & mf3xd3(y) := 0 \\ mf1yd3(z) := 4 - z & mf2zd3(x) := 0 & mf3zd3(y) := 0 \end{bmatrix}$$

Aplicación del método de la CUF:

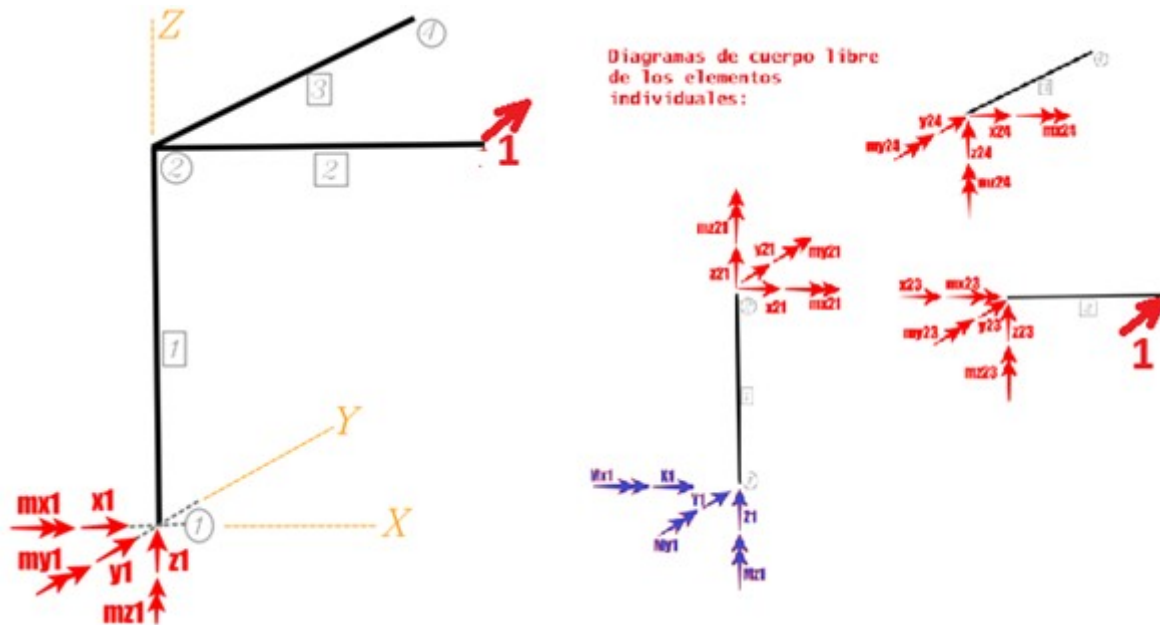
$$\begin{aligned} \#37: \delta_{3x} := & \frac{1}{E \cdot I_{1x}} \cdot \int_0^4 MF_{1x}(z) \cdot mf_{1xd3}(z) \, dz + \frac{1}{E \cdot I_{1y}} \cdot \int_0^4 MF_{1y}(z) \cdot mf_{1yd3}(z) \, dz + \\ & \frac{1}{E \cdot I_{2y}} \cdot \int_0^3 MF_{2y}(x) \cdot mf_{2yd3}(x) \, dx + \frac{1}{E \cdot I_{2z}} \cdot \int_0^3 MF_{2z}(x) \cdot mf_{2zd3}(x) \, dx + \frac{1}{E \cdot I_{3z}} \cdot \int_0^2 \\ & MF_{3z}(y) \cdot mf_{3zd3}(y) \, dy + \frac{1}{E \cdot I_{3x}} \cdot \int_0^2 MF_{3x}(y) \cdot mf_{3xd3}(y) \, dy \end{aligned}$$

$$\#38: \quad \delta_{3x} := \frac{1}{9000} \cdot 0 + \frac{1}{4000} \cdot \frac{584}{3} + \frac{2}{1125} \cdot 0 + \frac{1}{250} \cdot 0 + \frac{1}{250} \cdot 0 + \frac{2}{1125} \cdot 0$$

$$\#39: \quad \delta_{3x} := \frac{73}{1500}$$

$$\#40: \quad \delta_{3x} = 0.04866666666$$

Deformación en Y del nudo 3 con el método de la Carga Unitaria Ficticia:



$$\#41: \quad [x1 :=, y1 :=, z1 :=, mx1 :=, my1 :=, mz1 :=]$$

$$\#42: \quad \begin{bmatrix} x21 := & y21 := & z21 := & x23 := & y23 := & z23 := & x24 := & y24 := & z24 := \\ mx21 := & my21 := & mz21 := & mx23 := & my23 := & mz23 := & mx24 := & my24 := & mz24 := \end{bmatrix}$$

$$\#43: \quad \begin{bmatrix} x1 = 0 \\ y1 + 1 = 0 \\ z1 = 0 \\ mx1 - 1 \cdot 4 = 0 \\ my1 = 0 \\ mz1 + 1 \cdot 3 = 0 \end{bmatrix}$$

$$\#44: \quad [my1 := 0, mx1 := 4, mz1 := -3, x1 := 0, y1 := -1, z1 := 0]$$

Columna 1:

$$\#45: \quad \begin{bmatrix} x1 + x21 = 0 & y1 + y21 = 0 & z1 + z21 = 0 \\ mx1 + mx21 + y1 \cdot 4 & my1 + my21 - x1 \cdot 4 = 0 & mz1 + mz21 = 0 \end{bmatrix}$$

$$\#46: \quad [mx21 := 0, my21 := 0, mz21 := 3, x21 := 0, y21 := 1, z21 := 0]$$

Viga 2:

$$\#47: \quad \begin{bmatrix} x23 = 0 & y23 + 1 = 0 & z23 = 0 \\ mx23 = 0 & my23 = 0 & mz23 + 1 \cdot 3 = 0 \end{bmatrix}$$

$$\#48: \quad [mx23 := 0, my23 := 0, mz23 := -3, x23 := 0, y23 := -1, z23 := 0]$$

Viga 3=0

Momentos flectores "minúscula" de la estructura con cargas unitarias ficticias para desplazamiento en el nudo 3:

$$\#49: \quad \begin{bmatrix} mf1xd3(z) := mx1 + y1 \cdot z & mf2yd3(x) := my23 + z23 \cdot x & mf3xd3(y) := 0 \\ mf1yd3(z) := -my1 + x1 \cdot z & mf2zd3(x) := -mz23 + y23 \cdot x & mf3zd3(y) := 0 \end{bmatrix}$$

$$\#50: \quad \begin{bmatrix} mf1xd3(z) := 4 - z & mf2yd3(x) := 0 & mf3xd3(y) := 0 \\ mf1yd3(z) := 0 & mf2zd3(x) := 3 - x & mf3zd3(y) := 0 \end{bmatrix}$$

Aplicación del método de la CUF:

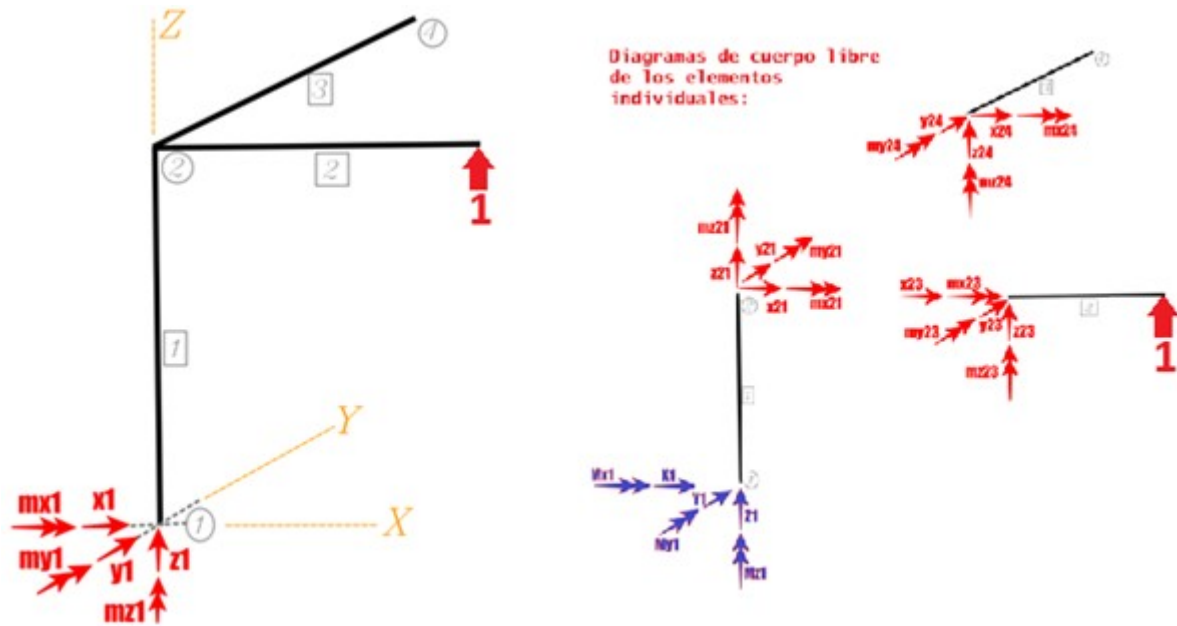
$$\begin{aligned} \#51: \quad \delta_{3y} := & \frac{1}{E \cdot I_{1x}} \cdot \int_0^4 MF1x(z) \cdot mf1xd3(z) \, dz + \frac{1}{E \cdot I_{1y}} \cdot \int_0^4 MF1y(z) \cdot mf1yd3(z) \, dz + \\ & \frac{1}{E \cdot I_{2y}} \cdot \int_0^3 MF2y(x) \cdot mf2yd3(x) \, dx + \frac{1}{E \cdot I_{2z}} \cdot \int_0^3 MF2z(x) \cdot mf2zd3(x) \, dx + \frac{1}{E \cdot I_{3z}} \cdot \int_0^2 \\ & MF3z(y) \cdot mf3zd3(y) \, dy + \frac{1}{E \cdot I_{3x}} \cdot \int_0^2 MF3x(y) \cdot mf3xd3(y) \, dy \end{aligned}$$

$$\#52: \quad \delta_{3y} := \frac{1}{9000} \cdot 240 + \frac{1}{4000} \cdot 0 + \frac{2}{1125} \cdot 0 + \frac{1}{250} \cdot 0 + \frac{1}{250} \cdot 0 + \frac{2}{1125} \cdot 0$$

$$\#53: \quad \delta_{3y} := \frac{2}{75}$$

$$\#54: \quad \delta_{3y} = 0.02666666666$$

Deformación en Z del nudo 3 con el método de la Carga Unitaria Ficticia:



#55: $[x1 :=, y1 :=, z1 :=, mx1 :=, my1 :=, mz1 :=]$

#56:
$$\begin{bmatrix} x21 := & y21 := & z21 := & x23 := & y23 := & z23 := & x24 := & y24 := & z24 := \\ mx21 := & my21 := & mz21 := & mx23 := & my23 := & mz23 := & mx24 := & my24 := & mz24 := \end{bmatrix}$$

#57:
$$\begin{bmatrix} x1 = 0 \\ y1 = 0 \\ z1 + 1 = 0 \\ mx1 = 0 \\ my1 - 1 \cdot 3 = 0 \\ mz1 = 0 \end{bmatrix}$$

#58: $[mx1 := 0, my1 := 3, mz1 := 0, x1 := 0, y1 := 0, z1 := -1]$

Columna 1:

#59:
$$\begin{bmatrix} x1 + x21 = 0 & y1 + y21 = 0 & z1 + z21 = 0 \\ mx1 + mx21 + y1 \cdot 4 & my1 + my21 - x1 \cdot 4 = 0 & mz1 + mz21 = 0 \end{bmatrix}$$

#60: $[mx21 := 0, my21 := -3, mz21 := 0, x21 := 0, y21 := 0, z21 := 1]$

Viga 2:

#61:
$$\begin{bmatrix} x23 = 0 & y23 = 0 & z23 + 1 = 0 \\ mx23 = 0 & my23 - 1 \cdot 3 = 0 & mz23 = 0 \end{bmatrix}$$

#62: $[mx23 := 0, my23 := 3, mz23 := 0, x23 := 0, y23 := 0, z23 := -1]$

Viga 3=0

Momentos flectores "minúscula" de la estructura con cargas unitarias ficticias para desplazamiento en el nudo 3:

$$\#63: \begin{bmatrix} mf1xd3(z) := mx1 + y1 \cdot z & mf2yd3(x) := my23 + z23 \cdot x & mf3xd3(y) := 0 \\ mf1yd3(z) := -my1 + x1 \cdot z & mf2zd3(x) := -mz23 + y23 \cdot x & mf3zd3(y) := 0 \end{bmatrix}$$

$$\#64: \begin{bmatrix} mf1xd3(z) := 0 & mf2yd3(x) := 3 - x & mf3xd3(y) := 0 \\ mf1yd3(z) := -3 & mf2zd3(x) := 0 & mf3zd3(y) := 0 \end{bmatrix}$$

Aplicación del método de la CUF:

$$\#65: \delta_{3z} := \frac{1}{E \cdot I_{1x}} \cdot \int_0^4 MF1x(z) \cdot mf1xd3(z) dz + \frac{1}{E \cdot I_{1y}} \cdot \int_0^4 MF1y(z) \cdot mf1yd3(z) dz +$$

$$\frac{1}{E \cdot I_{2y}} \cdot \int_0^3 MF2y(x) \cdot mf2yd3(x) dx + \frac{1}{E \cdot I_{2z}} \cdot \int_0^3 MF2z(x) \cdot mf2zd3(x) dx + \frac{1}{E \cdot I_{3z}} \cdot \int_0^2$$

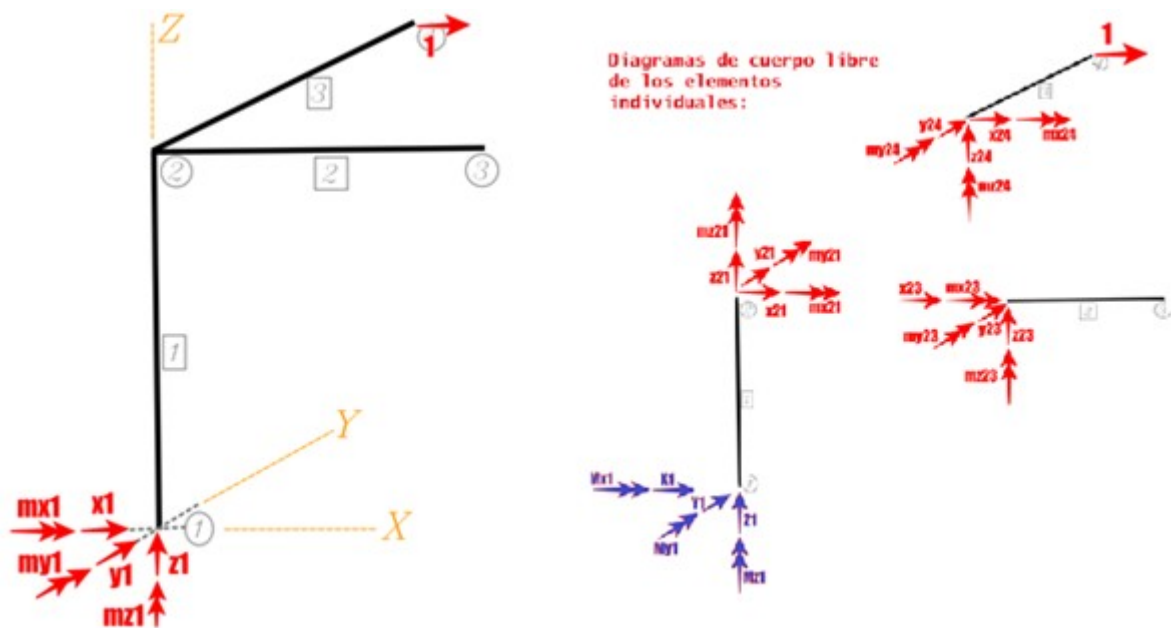
$$MF3z(y) \cdot mf3zd3(y) dy + \frac{1}{E \cdot I_{3x}} \cdot \int_0^2 MF3x(y) \cdot mf3xd3(y) dy$$

$$\#66: \delta_{3z} := \frac{1}{9000} \cdot 0 + \frac{1}{4000} \cdot (-220) + \frac{2}{1125} \cdot (-9) + \frac{1}{250} \cdot 0 + \frac{1}{250} \cdot 0 + \frac{2}{1125} \cdot 0$$

$$\#67: \delta_{3z} := -\frac{71}{1000}$$

$$\#68: \delta_{3z} = -0.071$$

Deformación en X del nudo 4 con el método de la Carga Unitaria Ficticia:



$$\#69: [x1 :=, y1 :=, z1 :=, mx1 :=, my1 :=, mz1 :=]$$

$$\#70: \begin{bmatrix} x21 := & y21 := & z21 := & x23 := & y23 := & z23 := & x24 := & y24 := & z24 := \\ mx21 := & my21 := & mz21 := & mx23 := & my23 := & mz23 := & mx24 := & my24 := & mz24 := \end{bmatrix}$$

$$\#71: \begin{bmatrix} x1 + 1 = 0 \\ y1 = 0 \\ z1 = 0 \\ mx1 = 0 \\ my1 + 1 \cdot 4 = 0 \\ mz1 - 1 \cdot 2 = 0 \end{bmatrix}$$

$$\#72: [mx1 := 0, my1 := -4, mz1 := 2, x1 := -1, y1 := 0, z1 := 0]$$

Columna 1:

$$\#73: \begin{bmatrix} x1 + x21 = 0 & y1 + y21 = 0 & z1 + z21 = 0 \\ mx1 + mx21 + y1 \cdot 4 & my1 + my21 - x1 \cdot 4 = 0 & mz1 + mz21 = 0 \end{bmatrix}$$

$$\#74: \begin{bmatrix} x1 + x21 = 0 & y1 + y21 = 0 & z1 + z21 = 0 \\ mx1 + mx21 + y1 \cdot 4 & my1 + my21 - x1 \cdot 4 = 0 & mz1 + mz21 = 0 \end{bmatrix}$$

Viga 2=0

Viga 3:

$$\#75: \begin{bmatrix} x24 + 1 = 0 & y24 = 0 & z24 = 0 \\ mx24 = 0 & my24 = 0 & mz24 - 1 \cdot 2 = 0 \end{bmatrix}$$

$$\#76: [mx24 := 0, my24 := 0, mz24 := 2, x24 := -1, y24 := 0, z24 := 0]$$

Momentos flectores "minúscula" de la estructura con cargas unitarias ficticias para desplazamiento en el nudo 4:

$$\#77: \begin{bmatrix} mf1xd3(z) := mx1 + y1 \cdot z & mf2yd3(x) := 0 & mf3xd3(y) := -mx24 + z24 \cdot y \\ mf1yd3(z) := -my1 + x1 \cdot z & mf2zd3(x) := 0 & mf3zd3(y) := mz24 + x24 \cdot y \end{bmatrix}$$

$$\#78: \begin{bmatrix} mf1xd3(z) := 0 & mf2yd3(x) := 0 & mf3xd3(y) := 0 \\ mf1yd3(z) := 4 - z & mf2zd3(x) := 0 & mf3zd3(y) := 2 - y \end{bmatrix}$$

Aplicación del método de la CUF:

$$\#79: \delta_{4x} := \frac{1}{E \cdot I_{1x}} \cdot \int_0^4 MF1x(z) \cdot mf1xd3(z) dz + \frac{1}{E \cdot I_{1y}} \cdot \int_0^4 MF1y(z) \cdot mf1yd3(z) dz +$$

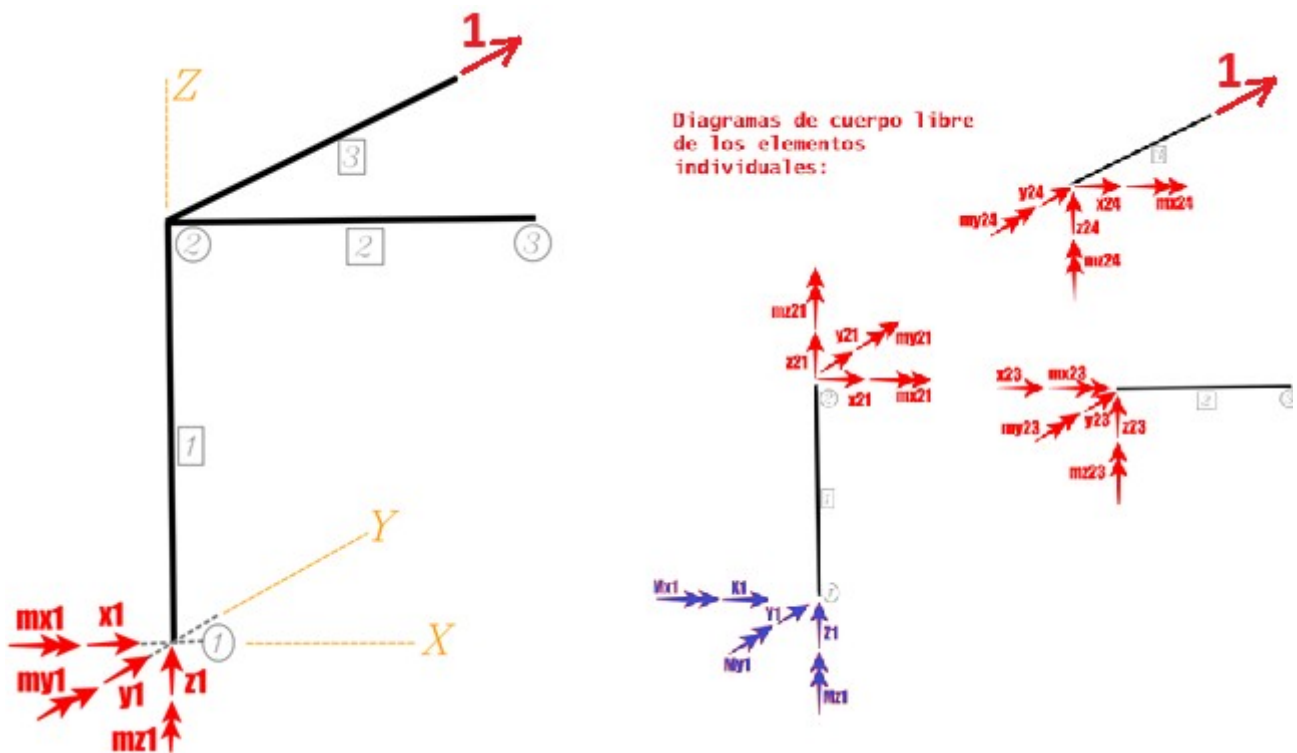
$$\frac{1}{E \cdot I_{2y}} \cdot \int_0^3 MF_{2y}(x) \cdot mf_{2y}d3(x) dx + \frac{1}{E \cdot I_{2z}} \cdot \int_0^3 MF_{2z}(x) \cdot mf_{2z}d3(x) dx + \frac{1}{E \cdot I_{3z}} \cdot \int_0^2 MF_{3z}(y) \cdot mf_{3z}d3(y) dy + \frac{1}{E \cdot I_{3x}} \cdot \int_0^2 MF_{3x}(y) \cdot mf_{3x}d3(y) dy$$

#80: $\delta 4x := \frac{1}{9000} \cdot 0 + \frac{1}{4000} \cdot \frac{584}{3} + \frac{2}{1125} \cdot 0 + \frac{1}{250} \cdot 0 + \frac{1}{250} \cdot 0 + \frac{2}{1125} \cdot 0$

#81: $\delta 4x := \frac{73}{1500}$

#82: $\delta 4x = 0.04866666666$

Deformación en Y del nudo 4 con el método de la Carga Unitaria Ficticia:



#83: $[x1 :=, y1 :=, z1 :=, mx1 :=, my1 :=, mz1 :=]$

#84: $\begin{bmatrix} x21 := & y21 := & z21 := & x23 := & y23 := & z23 := & x24 := & y24 := & z24 := \\ mx21 := & my21 := & mz21 := & mx23 := & my23 := & mz23 := & mx24 := & my24 := & mz24 := \end{bmatrix}$

$$\#85: \begin{bmatrix} x1 = 0 \\ y1 + 1 = 0 \\ z1 = 0 \\ mx1 - 1 \cdot 4 = 0 \\ my1 = 0 \\ mz1 = 0 \end{bmatrix}$$

$$\#86: [mx1 := 4, my1 := 0, mz1 := 0, x1 := 0, y1 := -1, z1 := 0]$$

Columna 1:

$$\#87: \begin{bmatrix} x1 + x21 = 0 & y1 + y21 = 0 & z1 + z21 = 0 \\ mx1 + mx21 + y1 \cdot 4 & my1 + my21 - x1 \cdot 4 = 0 & mz1 + mz21 = 0 \end{bmatrix}$$

$$\#88: [mx21 := 0, my21 := 0, mz21 := 0, x21 := 0, y21 := 1, z21 := 0]$$

Viga 2=0

Viga 3:

$$\#89: \begin{bmatrix} x24 = 0 & y24 + 1 = 0 & z24 = 0 \\ mx24 = 0 & my24 = 0 & mz24 = 0 \end{bmatrix}$$

$$\#90: [mx24 := 0, my24 := 0, mz24 := 0, x24 := 0, y24 := -1, z24 := 0]$$

Momentos flectores "minúscula" de la estructura con cargas unitarias ficticias para desplazamiento en el nudo 4:

$$\#91: \begin{bmatrix} mf1xd3(z) := mx1 + y1 \cdot z & mf2yd3(x) := 0 & mf3xd3(y) := -mx24 + z24 \cdot y \\ mf1yd3(z) := -my1 + x1 \cdot z & mf2zd3(x) := 0 & mf3zd3(y) := mz24 + x24 \cdot y \end{bmatrix}$$

$$\#92: \begin{bmatrix} mf1xd3(z) := 4 - z & mf2yd3(x) := 0 & mf3xd3(y) := 0 \\ mf1yd3(z) := 0 & mf2zd3(x) := 0 & mf3zd3(y) := 0 \end{bmatrix}$$

Momentos flectores "minúscula" de la estructura con cargas unitarias ficticias para desplazamiento en el nudo 4:

Aplicación del método de la CUF:

$$\begin{aligned} \#93: \delta 4y := & \frac{1}{E \cdot I1x} \cdot \int_0^4 MF1x(z) \cdot mf1xd3(z) \, dz + \frac{1}{E \cdot I1y} \cdot \int_0^4 MF1y(z) \cdot mf1yd3(z) \, dz + \\ & \frac{1}{E \cdot I2y} \cdot \int_0^3 MF2y(x) \cdot mf2yd3(x) \, dx + \frac{1}{E \cdot I2z} \cdot \int_0^3 MF2z(x) \cdot mf2zd3(x) \, dx + \frac{1}{E \cdot I3z} \cdot \int_0^2 \end{aligned}$$

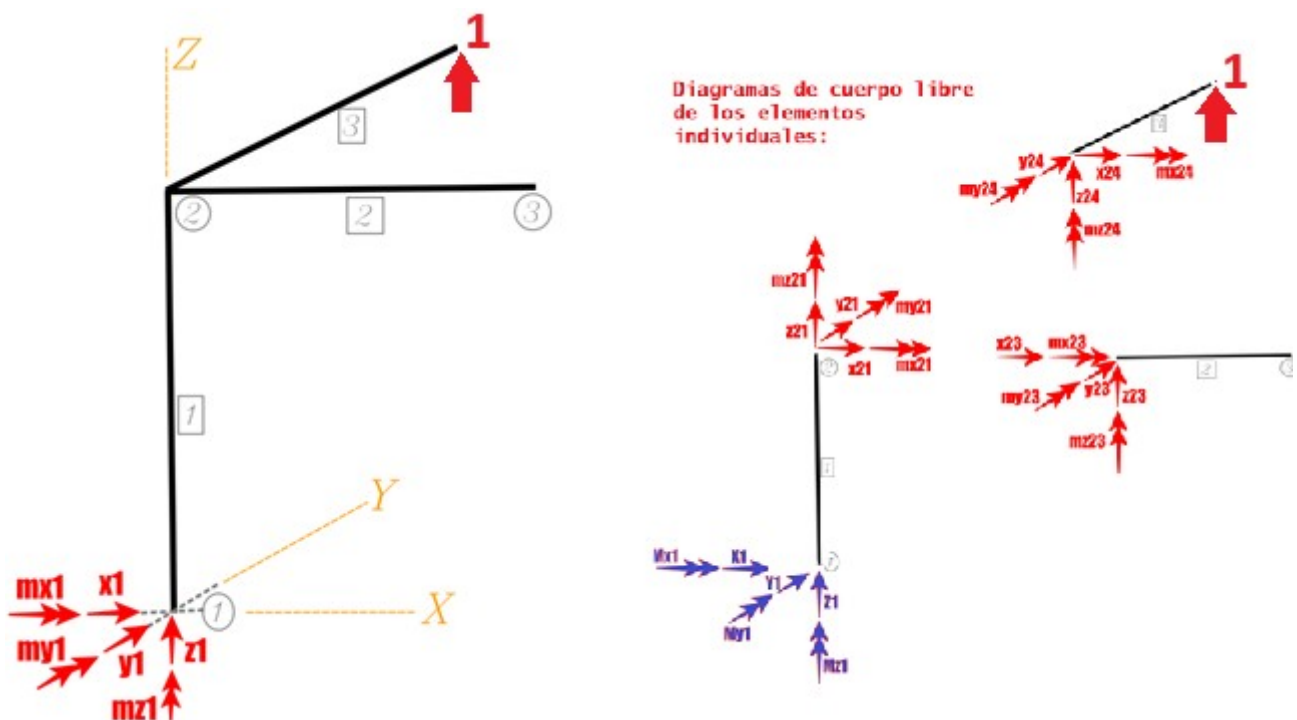
$$MF3z(y) \cdot mf3zd3(y) dy + \frac{1}{E \cdot I3x} \cdot \int_0^2 MF3x(y) \cdot mf3xd3(y) dy$$

#94: $\delta 4y := \frac{1}{9000} \cdot 240 + \frac{1}{4000} \cdot 0 + \frac{2}{1125} \cdot 0 + \frac{1}{250} \cdot 0 + \frac{1}{250} \cdot 0 + \frac{2}{1125} \cdot 0$

#95: $\delta 4y := \frac{2}{75}$

#96: $\delta 4y = 0.02666666666$

Deformación en Z del nudo 4 con el método de la Carga Unitaria Ficticia:



#97: $[x1 :=, y1 :=, z1 :=, mx1 :=, my1 :=, mz1 :=]$

#98: $\begin{bmatrix} x21 := & y21 := & z21 := & x23 := & y23 := & z23 := & x24 := & y24 := & z24 := \\ mx21 := & my21 := & mz21 := & mx23 := & my23 := & mz23 := & mx24 := & my24 := & mz24 := \end{bmatrix}$

#99: $\begin{bmatrix} x1 = 0 \\ y1 = 0 \\ z1 + 1 = 0 \\ mx1 + 1 \cdot 2 = 0 \\ my1 = 0 \\ mz1 = 0 \end{bmatrix}$

$$\#100: \quad [mx1 := -2, my1 := 0, mz1 := 0, x1 := 0, y1 := 0, z1 := -1]$$

Columna 1:

$$\#101: \quad \begin{bmatrix} x1 + x21 = 0 & y1 + y21 = 0 & z1 + z21 = 0 \\ mx1 + mx21 + y1 \cdot 4 & my1 + my21 - x1 \cdot 4 = 0 & mz1 + mz21 = 0 \end{bmatrix}$$

$$\#102: \quad [mx21 := 2, my21 := 0, mz21 := 0, x21 := 0, y21 := 0, z21 := 1]$$

Viga 2=0

Viga 3:

$$\#103: \quad \begin{bmatrix} x24 = 0 & y24 = 0 & z24 + 1 = 0 \\ mx24 + 1 \cdot 2 = 0 & my24 = 0 & mz24 = 0 \end{bmatrix}$$

$$\#104: \quad [mx24 := -2, my24 := 0, mz24 := 0, x24 := 0, y24 := 0, z24 := -1]$$

Momentos flectores "minúscula" de la estructura con cargas unitarias ficticias para desplazamiento en el nudo 4:

$$\#105: \quad \begin{bmatrix} mf1xd3(z) := mx1 + y1 \cdot z & mf2yd3(x) := 0 & mf3xd3(y) := -mx24 + z24 \cdot y \\ mf1yd3(z) := -my1 + x1 \cdot z & mf2zd3(x) := 0 & mf3zd3(y) := mz24 + x24 \cdot y \end{bmatrix}$$

$$\#106: \quad \begin{bmatrix} mf1xd3(z) := -2 & mf2yd3(x) := 0 & mf3xd3(y) := 2 - y \\ mf1yd3(z) := 0 & mf2zd3(x) := 0 & mf3zd3(y) := 0 \end{bmatrix}$$

Aplicación del método de la CUF:

$$\begin{aligned} \#107: \quad \delta_{4z} := & \frac{1}{E \cdot I1x} \cdot \int_0^4 MF1x(z) \cdot mf1xd3(z) \, dz + \frac{1}{E \cdot I1y} \cdot \int_0^4 MF1y(z) \cdot mf1yd3(z) \, dz + \\ & \frac{1}{E \cdot I2y} \cdot \int_0^3 MF2y(x) \cdot mf2yd3(x) \, dx + \frac{1}{E \cdot I2z} \cdot \int_0^3 MF2z(x) \cdot mf2zd3(x) \, dx + \frac{1}{E \cdot I3z} \cdot \int_0^2 \\ & MF3z(y) \cdot mf3zd3(y) \, dy + \frac{1}{E \cdot I3x} \cdot \int_0^2 MF3x(y) \cdot mf3xd3(y) \, dy \end{aligned}$$

$$\#108: \quad \delta_{4z} := \frac{1}{9000} \cdot (-208) + \frac{1}{4000} \cdot 0 + \frac{2}{1125} \cdot 0 + \frac{1}{250} \cdot 0 + \frac{1}{250} \cdot 0 + \frac{2}{1125} \cdot \left(-\frac{56}{3} \right)$$

$$\#109: \quad \delta_{4z} := -\frac{38}{675}$$

$$\#110: \quad \delta_{4z} = -0.05629629629$$

Resultados finales agrupados:

$$\#111: \begin{bmatrix} \delta_{3x} & \delta_{3y} & \delta_{3z} \\ \delta_{4x} & \delta_{4y} & \delta_{4z} \end{bmatrix} = \begin{bmatrix} \frac{73}{1500} & \frac{2}{75} & -\frac{71}{1000} \\ \frac{73}{1500} & \frac{2}{75} & -\frac{38}{675} \end{bmatrix}$$

$$\#112: \begin{bmatrix} \delta_{3x} & \delta_{3y} & \delta_{3z} \\ \delta_{4x} & \delta_{4y} & \delta_{4z} \end{bmatrix} = \begin{bmatrix} 0.04866666666 & 0.02666666666 & -0.071 \\ 0.04866666666 & 0.02666666666 & -0.05629629629 \end{bmatrix}$$